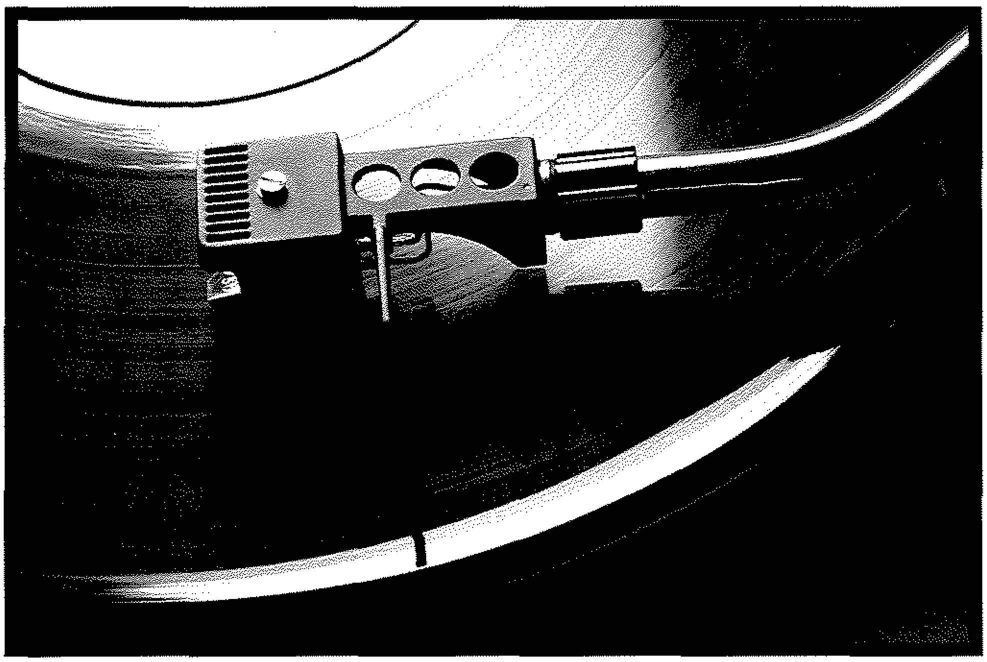
CHAPTER 10



The spiral groove of a record, from the outer rim to the inner

Coming in this chapter:

Exploration I have pennies, dimes, and quarters, and two bags to put them in. I put all the coins of one kind into one bag, and coins of the other two kinds into the other. There is the same number of coins in each bag, and the total value of each bag's contents is the same. How much money might I have?

SATISFYING CONSTRAINTS

LESSON 10.1

The Van Pool

You will need:

graph paper

In the town of Braun a group of people decided to organize a van pool to get to and from work and school. They estimated mileage costs to be about \$11 per day, so the total cost including the bridge toll would be \$12 per day round trip. Then they had to discuss how to share costs. They agreed that children and adults might have different fares.

1. **Exploration** Say there are three children and four adults in the van pool. Find several possible fares you could charge children and adults.

SHARING COSTS

In the following problems, let x stand for a child's daily fare, and let y stand for an adult's daily fare.

2. If only one adult and one child joined the van pool, there is more than one possible pair of values for x and y.
   a. List three possible (x, y) pairs.
   b. Plot these (x, y) pairs on coordinate axes.
   c. Make a graph showing all possible (x, y) pairs. Label the x-intercept and the y-intercept.

3. Repeat problem 2, assuming that one child and two adults join the van pool. Use the same axes for your graph.

4. Repeat problem 2, assuming that two adults and three children join the van pool. Use the same axes for your graph.

5. a. Write equations for the graphs you drew in problems 2-4.
   b. For each equation, interpret the coefficients of x and y and the constant term in terms of the situation.
   c. Find the x-intercept and the y-intercept on each of your graphs. Interpret them in terms of the situation.

NEGOTIATIONS

In this section, assume that the van pool has four children and three adults.

6. a. Make a table showing several possible (x, y) pairs representing the daily fare for children and adults. Draw a graph that shows all possible (x, y) pairs.
   b. Label and interpret the x-intercept and the y-intercept on your graph.
   c. Write an equation for the graph.

The members of the van pool discussed how to divide the cost among themselves. Some thought the adults' and children's fares should be different, and others thought they should be the same. They discussed several possible plans.

In each case described in problems 7-12:

a. Figure out what the daily fare for adults and for children would be. Show your work.

b. Plot a point on the graph from problem 6 to represent your solution.

7. Frances suggested that adults pay twice as much as children because they have more money.

8. John thought that adults should pay $1 more than children.

9. Kathleen said that adults should pay $2 more than children.

10. Joanna argued that there was no reason to have different fares, since an adult and a child each occupy one seat.

11. Allan thought it was unfair to have adults pay more than children, since adults take turns driving the van. He argued that children should pay twice as much as adults.

12. Louise remembered that van pools are exempt from the bridge toll, so she subtracted $1 from the total cost. She agreed with Allan that children should pay twice as much as adults.

INFLATION

The cost of commuting kept increasing. Since the van could legally carry nine people, including the driver, the members decided to let two more children join the pool. They had six children and three adults. Over the years, the cost went up, first to $14, then to $15, and finally to $18 per day.

13. On the same pair of axes, draw three graphs, one for each of the three values for the total cost.

14. Label each graph with its equation.

15. Assume that the adults' fare is twice the children's fare. Mark the points on your graph representing those fares for adults and children, if the total cost is the following amounts:

a. $14 b. $15 c. $18

16. Look at the three points you marked in problem 15. You should be able to connect all of them with a straight line.

a. Find an equation that fits your line.

b. Interpret your equation. (What do the coefficients mean in terms of the problem?)

17. Repeat problems 15-16, assuming that the children's fare is twice the adults' fare.

REVIEW/PREVIEW *RECIPES*

These are the instructions on a can of orange juice concentrate.

> Mix one part juice concentrate
> with three parts water.

18. How much concentrate should you use to make

a. 6 cups of orange juice?

b. 10 cups of orange juice?

19. Using this recipe, how much of each ingredient would you need to make 160 cups of punch for the 80 people who are expected at the piano recital?

> **Piano Recital Punch**
>
> *Mix:*
>
> 4 parts iced tea, sweetened
> 4 parts apple juice
> 4 parts cranberry juice
> 2 parts orange juice
> 1 part lemon juice
>
> *Garnish with lemon and orange slices.*

20. How much Piano Recital Punch could you make if you had an unlimited amount of the other ingredients but only

a. 3/4 cup of lemon juice?

b. 3 cups of orange juice?

REVIEW *EXPONENTS*

21. Write without parentheses.

a. $(4x^2)^3$ b. $(4x^2y)^3$

22. Simplify each ratio.

a. $\frac{80 \cdot 2^{x+2}}{4 \cdot 2^x}$

b. $\frac{4 \cdot 2^{x+2}}{80 \cdot 2^{x+1}}$

c. $\frac{4 \cdot 2^{x+1}}{80 \cdot 2^{x+2}}$

23. Use your calculator to compare $3 \cdot 2^x$ and $2 \cdot 3^x$. Which is greater for different values of x? For what value of x are they equal?

LESSON 10.2

How Much of Each Kind?

AT THE LAUNDROMAT®

1. **Exploration** Some dimes and quarters have a total value of \$3.95. How many of each coin might there be? (Find all the possibilities.) What is the fewest coins there could possibly be? The most? Explain, showing your method of thinking about this problem and commenting on any patterns you notice.

Dan needs nickels and quarters to do his laundry at Science and Math Quick Wash. He has a five-dollar bill. The table shows one possible combination of coins he might get if he asks for change in nickels and quarters only. (The value is given in cents.)

Nickels		Quarters		Total Coins	
no.	**value**	**no.**	**value**	**no.**	**value**
45	225	11	275	56	500

2. Add at least six more possibilities to the table and comment on any patterns you notice. (If you don't see any patterns, add more possibilities until you do.)

3. What is the fewest coins Dan might get? The most?

4. Would it be possible for Dan to have an even number of coins? An odd number? Explain.

5. Would it be possible for Dan to have the same number of quarters as nickels? If so, how many of each would he have?

If Dan gets x nickels and y quarters, the entry in the table would look like this.

Nickels		Quarters		Total Coins	
no.	**value**	**no.**	**value**	**no.**	**value**
x	$5x$	y	$25y$	—	—

6. a. Explain the meaning of the expressions $5x$ and $25y$ in the table.
   b. Complete the entry, giving the total number of coins and their value *in terms of x and y.*

Any possible whole number pair of values (x, y) giving a possibility for the number of nickels and quarters that Dan might get in change will satisfy this *equation*,

$$5x + 25y = 500.$$

For example, it is easy to show by substitution that the pair (45, 11) satisfies this equation: $5(45) + 25(11) = 500$. This pair also satisfies this condition, or *constraint:*

The total number of coins is 56.

7. Is there another (x, y) pair that satisfies the same equation and the same constraint? If so, what is it?

8. Find (x, y) pairs that satisfy both the equation $5x + 25y = 500$ and the constraints given. (You may want to extend the table you made. You can save work by looking for patterns in your table.) Some may not be possible.
   Constraints:
   a. The total number of coins is 80.
   b. There are 20 times as many nickels as quarters.
   c. There are 12 more nickels than quarters.
   d. There are 8 more quarters than nickels.

9. Each of the constraints in problem 8 can be expressed as an equation in x and y. Write each equation.

10. At Science and Math Quick Wash, the machines take three quarters and one nickel to wash and one quarter to dry. If Dan wants to do as many loads as possible,
    a. how many loads of wash will he be able to do?
    b. what change should he request for his five-dollar bill? (Find all possible answers.)

CRANBERRY-APPLE JUICE

Nelson works for the G. Ale Bar Company, a chain of soda fountains. He is trying to create a special recipe to make a best-selling juice. For a taste test, he wants to prepare several 20-cup batches of different mixtures of pure apple juice with a cranberry-apple juice that is 50% apple and 50% cranberry.

Apple juice		**Cranberry-apple**		**Mixture**	
apple	**cran**	**apple**	**cran**	**apple**	**cran**
15	0	2.5	2.5	17.5	2.5
8	0	6	6	14	6
6	0	—	—	—	—
—	—	8	—	—	—
—	—	—	9.5	—	—
x	—	$0.50y$	—	$x+0.50y$	—

11. Copy and complete the table. Add several more numerical possibilities. (The last row is based on x cups of apple juice and y cups of cranberry-apple.)

12. What are the largest and smallest amounts of cranberry juice possible in one of Nelson's mixtures? What about apple juice? Explain.

13. For the mixture in the first row, what percent *of the total* is cranberry? (Be careful! You can't get the answer by dividing 2.5 by 17.5)

14. Add a **% cran** column to your table, and repeat problem 13 for all the rows.

15. What are the largest and smallest *percentages* of cranberry juice possible in one of Nelson's mixtures? What about apple juice? Explain.

16. Explain why the expression $x + 0.50y$ represents the total amount of apple juice in the mixture.

17. What is the expression for the total amount of cranberry juice in the mixture? Explain.

18. Explain why $x + y$ equals 20 for every possibility listed in the table.

19. What does the $x + 0.50y = 10$ mean in this situation? Is there an (x, y) pair that satisfies this equation? Explain.

For each equation, 20-25:

a. Interpret the equation in terms of this situation.
b. If possible, find a value of x and of y that satisfies the equation, given the constraint that x and y add up to 20.

20. $0.50y = 4$

21. $x + 0.50y = 15$

22. $x + 0.50y = 4$

23. $x + 0.5y = 11.5$

24. $x + 0.50y = 0.75(x + y)$

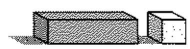
25. $x + 0.50y = 0.25(x + y)$

26. Which of the equations 20-25 were impossible to solve? Would they have been possible if the total amount had been 30 cups? Explain.

LESSON 10.3

Two Variables, Two Equations

You will need:

the Lab Gear 

MYSTERY CONTAINERS

The Lab Gear may help you solve this problem.

1. A crate contains two small containers and three large containers. The total weight of the crate is 16 pounds.

a. What are some possible weights of the small and the large containers? How many possible weights are there?

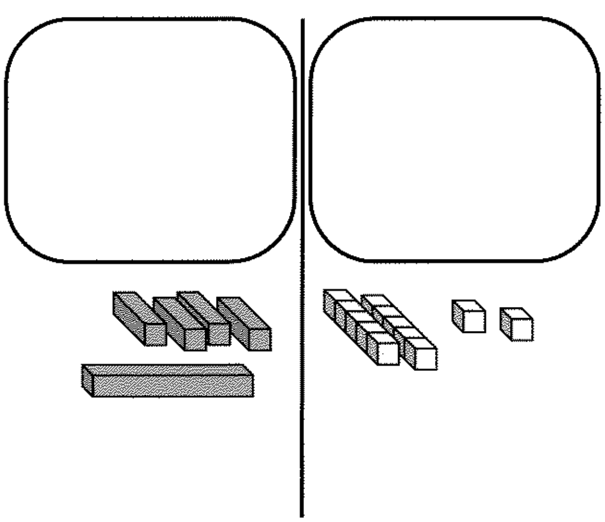
b. Find the weight of four small containers and six large containers.

c. Two containers are removed from the crate, and it is weighed again. Now it weighs ten pounds. Using this additional information, find possible weights for the small container and the large container. Comment on your answers.

ONE EQUATION, ONE CONSTRAINT

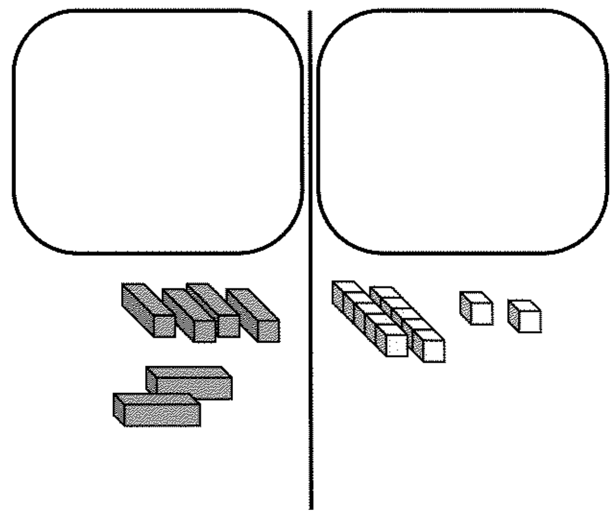
The workmat shows the equation

$$y + 4x = 12.$$



2. Using trial and error, find some values of x and y that make the equation true. (How many possible values are there?)

One of the (x, y) pairs satisfying this equation also satisfies the constraint, or condition, that y is twice x. If y is twice x, then each y-block can be replaced with two x-blocks.



The resulting equation is $6x = 12$.

3. Solve for x in the equation above. Then find the (x, y) pair that satisfies both the equation $y + 4x = 12$ and the constraint that $y = 2x$.

For each problem, 4-7, model the equation on the workmat with the blocks. Then use the blocks to find an (x, y) pair that satisfies both the equation and the constraint. Check your final answers in the original equations.

4. $4x - 7 = y + 3$
Constraint: y is two more than x.

5. $2y + x = 5$
Constraint: x is six less than y.

6. $2x + y = 9$
Constraint: x is three more than y.

7. $2y + x = 4$
Constraint: x and y add up to six.

For each problem you just solved, the constraint could have been written as an equation. For example, the constraint that the sum of x and y is six can be written $x + y = 6$. This means that in each of problems 4-7, you found an (x, y) pair that *satisfied both of two given equations*. We say that you *solved* a system of simultaneous equations.

SIMULTANEOUS EQUATIONS

Solve each system of simultaneous equations. If you want to use the Lab Gear, begin by modeling the first equation with the blocks. Then use the second equation to substitute blocks for the y-blocks or for the x-blocks. Check your answers.

8. $\begin{cases} 2x - y = 2 \\ y = 3x \end{cases}$

9. $\begin{cases} 4x + y = 10 \\ y = 6x - 20 \end{cases}$

10. $\begin{cases} x - 4y = 23 \\ x = -5y - 4 \end{cases}$

11. $\begin{cases} 3y + 2x = 7 \\ 3y = 4x - 5 \end{cases}$

MIND READING

What numbers am I thinking of?

12. Their sum is 7. Their difference is 3.

13. Their sum is 18. The second is twice as large as the first.

14. The first minus the second is 3. Twice the first, minus twice the second is 6.

15. One of problems 12-14 has more than one answer. How many answers does it have? Why?

REVIEW/PREVIEW *EVALUATING*

16. Two (x, y) pairs that satisfy the equation $2x + 3y = 16$ are given in the table below. Copy and complete the table.

x	y	$2x + 3y$	$x + y$	$x - y$	$4x + 6y$	$x + 1.5y$
-1	6	16	5	—	—	8
2	4	16	—	-2	32	—
—	5	16	—	—	—	—
—	-6	16	—	—	—	—
—	—	16	—	—	—	—
-4	—	16	—	—	—	—

17. Study the table you made. In which columns are all the values the same? Why?

REVIEW/PREVIEW *SOLVING FOR y*

Set up these problems with the Lab Gear, and rearrange the blocks so that y is by itself on one side of the equation. Write equations to show your steps. In some cases, you will need to finish the problem without the blocks.

18. $-4x + y = 6$

19. $4x + 2y = 10$

20. $-6x + y = 4$

21. $-6x + 3y = 9$

22. $6x - 3y = 12$

23. $x + 2y = 8$

24. $x - y = 1$

25. $6x - 5y = 0$

26. Explain how to solve for y (without the Lab Gear), with the help of an example.

27. **Generalization** Solve for y.

$$Ax + By = C$$

DISCOVERY *CONSECUTIVE NUMBERS*

28. Compute, and look for a pattern.

a. $1 \cdot 2 \cdot 3 + 2$

b. $2 \cdot 3 \cdot 4 + 3$

c. $4 \cdot 5 \cdot 6 + 5$

d. $(5 - 1) \cdot 5 \cdot (5 + 1) + 5$

e. $9 \cdot 10 \cdot 11 + 10$

f. $(10 - 1) \cdot 10 \cdot (10 + 1) + 10$

29. Use algebra to explain the pattern.

30. The product of three consecutive numbers divided by their sum is 1. What are the numbers?

31. Repeat problem 30, if the product divided by the sum is the following:

a. 5 b. 16

32. What can you say about the middle number if the product of three consecutive numbers divided by their sum is a whole number?

DISCOVERY *NINE FACTORS*

33. Project The number 1 has one whole number factor, itself; 2 has two factors, 1 and 2; 3 has two factors; and 4 has three factors. (What are they?) Find some numbers having *nine factors*. Explain.

LESSON 10.4
Solving Systems

You will need:

the Lab Gear 

Definitions: In real-world applications we often need to find a solution that satisfies two or more equations simultaneously. We call the group of equations a *system of simultaneous equations*. To *solve a system* means to find the (x, y) pairs that satisfy every equation in the group.

In this course, you will learn techniques for solving systems of two equations. In later courses you will learn how to solve systems of more than two equations.

In an earlier chapter, you studied equivalent equations. *Equivalent equations have all the same solutions.*

1. Find some (x, y) solutions to these equations.

a. $y = 2x + 6$

b. $3y = 6x + 18$

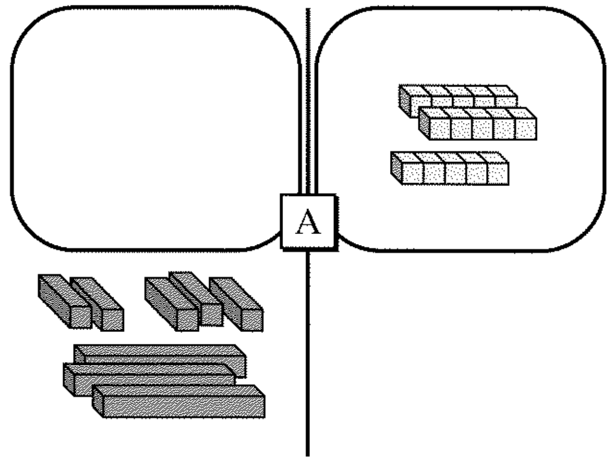
2. Use algebra to show that the two equations in problem 1 are equivalent.

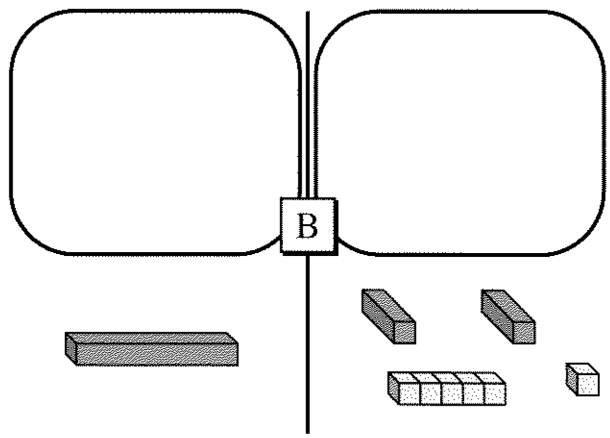
SOLVING TECHNIQUES: SUBSTITUTION

Example: Solve the system.

$$\begin{cases} 5x + 3y = -15 & \text{(A)} \\ y = 2x + 6 & \text{(B)} \end{cases}$$

The figure shows how to model the system on two workmats.

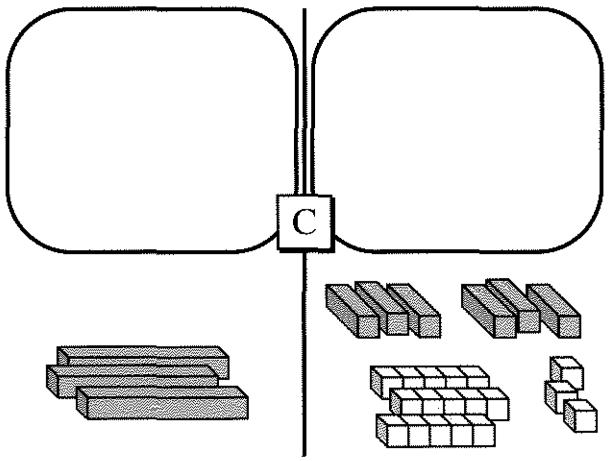




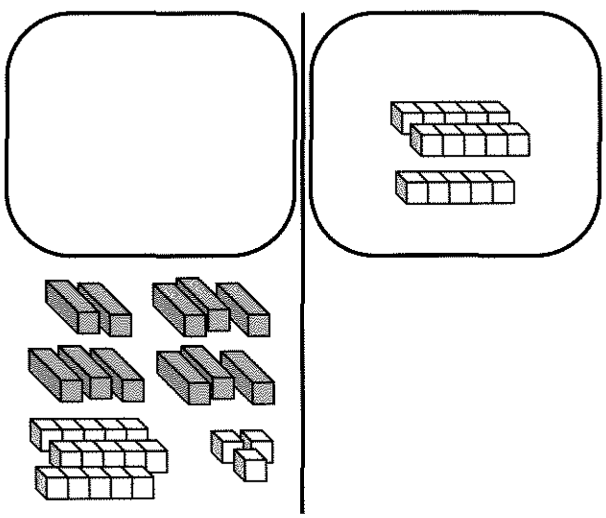
By multiplying both sides of Equation (B) by 3, you get Equation (C), which is equivalent to Equation B.

$$3y = 6x + 18 \quad \text{(C)}$$

The figure shows how to model Equation (C) on the second workmat.



Since $3y = 6x + 18$, we can replace the $3y$ in the first equation with $6x + 18$ to get a new equation that has only x-blocks and yellow blocks.



3. Write the new equation. Then solve for x.
4. a. Substitute the value of x into Equation (B) and solve for y.
   b. Substitute this (x, y) pair into Equation (A). If it doesn't satisfy the equation, check your work to find your mistake.
   c. Write the (x, y) pair that is the solution to the system.

Solve each system, 5-10. If you use the Lab Gear, you may set up the first equation with the blocks. Then use the second equation to eliminate the x- or y-blocks by substitution. In some cases, you may first need to write an equation equivalent to the second equation.

5. $\begin{cases} 5y - 4x = -9 \\ 5y = 3x - 7 \end{cases}$
6. $\begin{cases} 5x + 3y = -15 \\ y = 2x + 6 \end{cases}$
7. $\begin{cases} 5x - 3y = -29 \\ x = 2 - 2y \end{cases}$
8. $\begin{cases} 2x + 3y = 9 \\ 4x = 6 - 2y \end{cases}$
9. $\begin{cases} 4x - y = 5 \\ 3y = 6x + 3 \end{cases}$
10. $\begin{cases} 6x - 2y = -16 \\ 4x + y = 1 \end{cases}$

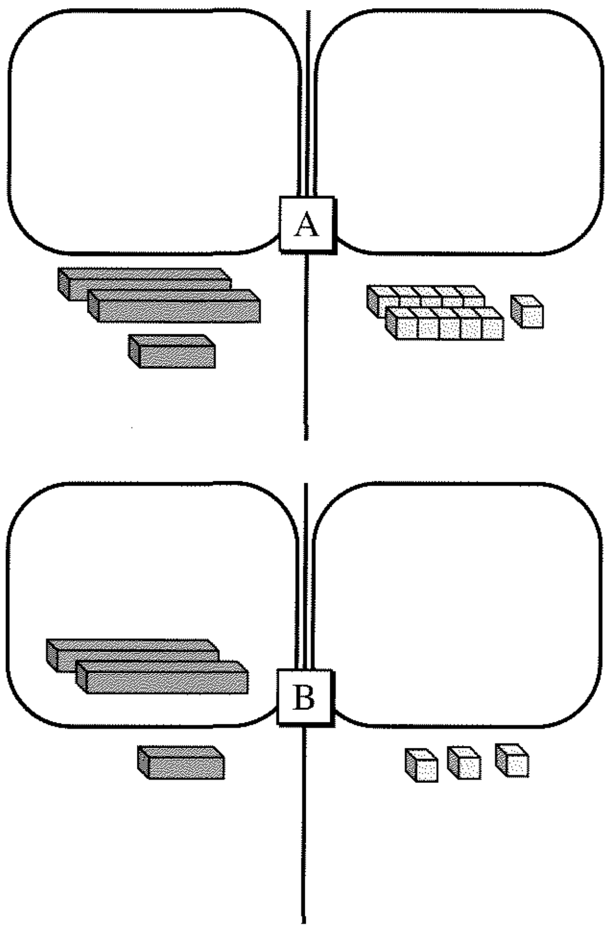
SOLVING TECHNIQUES: LINEAR COMBINATIONS

Here is another technique for solving systems.

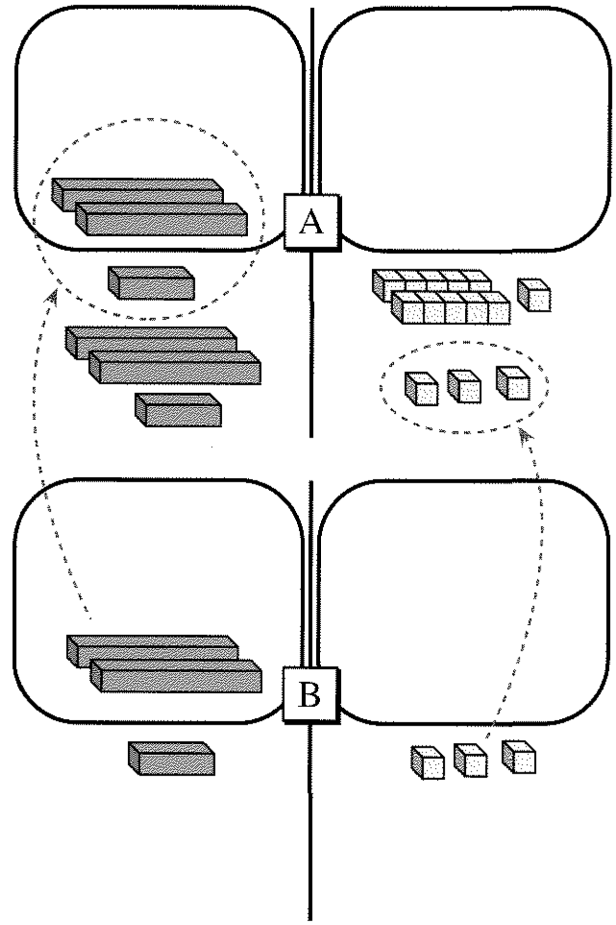
Example: Solve the system.

$$\begin{cases} x + 2y = 11 & \text{(A)} \\ x - 2y = 3 & \text{(B)} \end{cases}$$

The figure shows two workmats, with one equation modeled on each.



You can add equal quantities to both sides of Equation (A) to get an equivalent equation. For example, you could add 3 to both sides, or even $x - 2y$ to both sides. Also, since Equation (B) says that $x - 2y = 3$, you could add 3 to one side and $x - 2y$ to the other side, as shown on the figure.



11. Write the equation shown in equation (A) in the figure. Simplify and solve for x. (What happened to y?)

12. Find the (x, y) pair that is the solution to the system. Check by substituting into both of the original equations.

Solving the system in the example was easier than solving most systems, since when you added one equation to the other there were no y's left. The next example is more difficult.

Example: Solve the system.

$$\begin{cases} 2y - 6x = 16 & \text{(A)} \\ 4x + y = 1 & \text{(B)} \end{cases}$$

By multiplying both sides of Equation (B) by -2, you get Equation (C), which is equivalent to Equation (B). Here is the new system, which is equivalent to the original.

$$\begin{cases} 2y - 6x = 16 & \text{(A)} \\ -8x - 2y = -2 & \text{(C)} \end{cases}$$

13. Why was Equation (B) multiplied by -2?

14. Solve the system. Show your work. Check your answers by substituting into both equations of the original system.

Mr. Richards gave the class this hard system to solve.

$$\begin{cases} 3x + 5y = 17 & \text{(A)} \\ 2x + 3y = 11 & \text{(B)} \end{cases}$$

Charlotte suggested multiplying the first equation by 3 and the second equation by -5 to get a new system.

15. Use Charlotte's method to write a new system. Solve the system and check your answer.

Definition: The equation you get by adding multiples of the two equations together is called a *linear combination* of the two equations.

Leroy thought it would be easier if they got a linear combination by multiplying by smaller numbers. He suggested multiplying the first equation by -2 and the second equation by 3.

16. Use Leroy's method to write a new system. Solve the system.

17. Compare the two ways you solved this problem. Which do you prefer? Can you think of a third way? Explain.

SYSTEMATIC PRACTICE

Solve these systems. Some have one (x, y) solution. Others have an infinite number of solutions, or no solution.

18. $\begin{cases} 5x + 7y = 1 \\ x + 7 = 1 \end{cases}$

19. $\begin{cases} 3 - x = 4y \\ x = -2y - 9 \end{cases}$

20. $\begin{cases} 8x - 4y = 0 \\ 2x = y \end{cases}$

21. $\begin{cases} y = 4 + x \\ y = 7x + 10 \end{cases}$

22. $\begin{cases} 4x - y = 2 \\ y = 4x + 1 \end{cases}$

23. $\begin{cases} 6x - 2y = -16 \\ 4x + y = 1 \end{cases}$

THINKING WRITING

10.A Juice Experiments

Nelson is continuing his quest for the perfect juice. You have been hired as a consultant to the G. Ale Bar Company to assist him. He ran out of apple juice and is making the 20-cup batches for the taste test using two kinds of juice.

Fruity Flavor: 50% cranberry and 50% apple

Berry Blend: 20% cranberry and 80% apple

Fruity Flavor		Berry Blend		Mixture	
apple	**cran**	**apple**	**cran**	**apple**	**cran**
5	5	8	2	13	7
7.5	7.5	4	1	11.5	8.5
$0.50x$	—	$0.80y$	—	—	—

1. Make a table like the one above. List at least six possible mixtures. Add two columns to the table, showing the percents of cranberry and apple in the mixture.
2. Find the minimum and the maximum amount of cranberry juice possible in one of Nelson's mixtures. Then find the minimum and the maximum *percent*.
3. Repeat problem 2 for apple juice.

How many cups of Fruity Flavor and Berry Blend would you need to use to make 20 cups each of the cranberry-apple mixtures in 4-7? (Some are impossible.)

4. 30% cranberry, 70% apple
5. 25% cranberry, 75% apple
6. 10% cranberry, 90% apple
7. Choose your own percentages.
8. In the last line of the table, what is the meaning of x and y?
9. For Nelson's mixtures, what is the sum of x and y?

For each equation, 10-15:

a. Write, in words, an interpretation of it in terms of the situation.
b. If possible, find a value of x and of y that satisfies the equation, keeping in mind the answer to problem 9.

10. $0.50x + 0.20y = 7$
11. $0.50x + 0.80y = 8$
12. $0.50x + 0.80y = 0.25(x + y)$
13. $0.50x + 0.20y = 0.25(x + y)$
14. $x + y = 25$
15. $x - y = 10$
16. Report Write an illustrated report summarizing the results of this investigation. Your report should include, but not be limited to, answers to the following questions:
    - What determines the maximum and the minimum amount of each kind of juice possible in the mixture?
    - What determines the maximum and the minimum percent of each kind of juice in the mixture?
    - How could you use systems of equations to solve problems like 4 through 6? Give examples.

LESSON 10.5 Standard Form

You will need:
- graph paper
- graphing calculator (optional)

Definitions: An equation of the form $Ax + By = C$ is called the *standard form* of a linear equation. A, B, and C are the *parameters* for the equation.

In this lesson you will investigate how the values of the parameters affect the graphs of linear equations in standard form.

INTERCEPTS

Do not use graphing calculators for this section. These equations of lines are in standard form. For each equation:

a. Find the parameters A, B, and C.
b. Find the x-intercept and the y-intercept.
c. Graph the line by plotting the intercepts.

1. $3x + 2y = 12$ **2.** $3x - 2y = 18$

3. $x + y = 6$ **4.** $x - y = 6$

5. $-3x + 4y = 10$

6. **Generalization**

a. Explain how to find the x-intercept and the y-intercept of the line whose equation is $Ax + By = C$.

b. A fast way to graph a line is by finding and plotting the intercepts. Show how to use this technique to graph a line of the form $Ax + By = C$. (Choose specific values for A, B, and C.)

7. a. Write the equation of a line that has x-intercept (6, 0). Graph it and find its y-intercept.

b. Write the equation of a line that has y-intercept (0, -4). Graph it and find its x-intercept.

c. Write the equation of a line that has y-intercept (0, 4) *and* x-intercept (-6, 0).

8. **Generalization** Show how to find the equation of a line having intercepts $(p, 0)$ and $(0, q)$.

THE CASE WHEN $A = B$

9. a. Graph $x + y = 10$.

b. On the same axes, graph $2x + 2y = 10$.

c. In the equations you graphed in parts (a) and (b), what are A, B, and C?

d. When you doubled A and B in the equation but left C the same, how did the graph change?

10. Draw the graphs of at least two other equations of the form $Ax + By = C$ for which A is equal to B and $C = 10$. Label the graphs with their equations.

11. Compare all the graphs you drew in problems 9-10. (What stayed the same, and what changed? How do the graphs compare in steepness?)

12. a. Graph $x + y = 4$.

b. On the same axes, graph $2x + 2y = 8$.

c. In the equations you graphed in parts (a) and (b), what are A, B, and C?

13.

a. When you doubled A, B, and C, how did the graph change?

b. If you triple A, B, and C, what will the equation be? How do you think the graph will change? Explain.

VARYING A

14. a. Graph $x + 2y = 5$.
b. Graph $2x + 2y = 5$ on the same axes.
c. Draw several more graphs, changing the value of A, leaving B equal to 2, and C equal to 5. Use both positive and negative values for A.

15. Compare all the graphs you drew in problem 14.
a. When you changed the value of A in the equation, what features of the graph changed and what stayed the same? Did the steepness change? Did the intercepts change?
b. How are the graphs having a positive value of A different from the graphs having a negative value of A?
c. Is it possible to pick a value of A so that the graph will be a horizontal line? A vertical line? Explain.

16. Show what you think the following graphs would look like. You don't have to graph them accurately, but you should make a rough sketch and explain your work.
a. $500x + 2y = 5$
b. $-500x + 2y = 5$
c. $0.01x + 2y = 5$
d. $-0.01x + 2y = 5$

VARYING B

17. a. Graph $2x + y = 8$.
b. Graph $2x + 2y = 8$ on the same axes.
c. Draw several more graphs, changing the value of B, leaving A equal to 2, and C equal to 8. Use both positive and negative values for B.

18. Compare all the graphs you drew in problem 17.
a. When you changed the value of B in the equation, what features of the graph changed and what stayed the same? Did the steepness change? Did the intercepts change?
b. How are the graphs having a positive value of B different from the graphs having a negative value of B?
c. Is it possible to pick a value of B so that the graph will be a horizontal line? A vertical line? Explain.

19. Show what you think the following graphs would look like. You don't have to graph them accurately, but you should make a rough sketch and explain your work.
a. $2x + 100y = 8$
b. $2x - 100y = 8$
c. $2x + 0.02y = 8$
d. $2x - 0.02y = 8$

VARYING C

20. Where do you think the graph of $3x + 2y = 5$ will intersect the graph of $3x + 2y = 6$? You may want to check your prediction by graphing.

21. Describe what will happen to the graph of $3x + 2y = 6$ when you change the value of C but keep A and B constant. What will change and what will stay the same? Make several graphs to convince yourself that your answers are correct.

22. Report Write a report summarizing what you learned in this lesson. Explain how the values of the parameters A, B, and C affect the graph of $Ax + By = C$, specifically its slope and intercept. Use examples.

DISCOVERY ***DIFFERENCES OF PERFECT SQUARES***

23. **Project** The number 17 can be written as the difference of the squares of whole numbers, $9^2 - 8^2$. Which other whole numbers can be written as the difference of two squares of whole numbers? Which cannot? Look for patterns, and try to explain what you discover.

REVIEW ***SIDES OF SQUARES***

24. The length of a side of a square is given. Find the area of the square.

a. $\sqrt{2}$
b. $2 + \sqrt{2}$
c. $2 - \sqrt{2}$
d. $2\sqrt{2}$
e. $\sqrt{2}/2$
f. $2/\sqrt{2}$

25. The side lengths of two squares are given. Which of the two squares has the larger area? Explain how you know.

a. $\sqrt{10} - \sqrt{5}$ and $\sqrt{5}$
b. $2\sqrt{8}$ and $\sqrt{16}$

26. Which has the larger area, or are they the same?

a. a rectangle with sides $\sqrt{2}$ and $\sqrt{5}$ or a square with side $\sqrt{10}$
b. a rectangle with sides $\sqrt{4}$ and $\sqrt{8}$ or a square with side $2\sqrt{2}$

27. Which has the larger perimeter, or are they the same?

a. a rectangle with sides $\sqrt{10}$ and $\sqrt{5}$ or a square with side $2\sqrt{5}$
b. a rectangle with sides $2 + 2\sqrt{2}$ and $\sqrt{2}$ or a square with sides $2 + \sqrt{2}$

LESSON 10.6

Line Intersections

You will need:

- graph paper
- graphing calculator (optional)

POINTS ON LINES

1. On the same pair of axes, make accurate graphs of these three equations.

a. $3x + 5y = 9$ b. $6x + y = 18$

c. $4x + 2y = 30$

2. There is a point on each of the lines in problem 1 where the y-value is three times the x-value.

a. Find these points. Show your work.

b. The three points you found in part (a) should all lie on one straight line. What is the equation of this line?

3. Graph the line $4x + 2y = 6$. Then mark and label a point on the line for which

a. the y-coordinate is four times the x-coordinate;

b. y is twice x;

c. x is three less than y;

d. y is three less than x.

4. Add the graphs of the following lines to the axes you used in problem 3. Notice where each one intersects the line $4x + 2y = 6$.

a. $y = 4x$ b. $y = 2x$

c. $x = y - 3$ d. $y = x - 3$

5. Find the point on the line $2x - y = 6$ for which

a. the y-coordinate is one more than the x-coordinate;

b. the x-coordinate is 2/3 of the y-coordinate.

6. Explain the method you used to solve problem 5.

HOW MANY INTERSECTIONS?

7. Graph these three lines on the same pair of axes. Describe what you observe.

a. $x + 3y = 9$ $-x/3 + \frac{9}{3}$

b. $2x + 6y = 18$

c. $x + 3y = 10$

8. Graph the line $2x - 3y = 4$. Then write an equation that has

a. the same graph;

b. a parallel graph.

For each pair of equations 9-12 tell whether the two graphs will be

a. the same graph;

b. parallel graphs;

c. intersecting graphs.

9. $2x + 9 = y$
$-4x - 18 = -2y$

10. $x - y = 7$
$x + y = 7$

11. $x + 6 = y$
$x + y = 6$

12. $x + y = 9$
$x + y = 7$

13. **Summary** Explain how to tell without graphing whether the equations of two lines have the same graph, parallel graphs, or intersecting graphs. Give examples.

HOW MANY SOLUTIONS?

Some pairs of equations 14-19 represent parallel lines. Some represent intersecting lines. Others represent the same line. *Without graphing*, find the point of intersection of each pair of lines, if it exists.

14. $2x - 3y = 7$
$3x - 4y = 15$

15. $x = 6 + 3y$
$3y = 3 + x$

16. $y - 12 = 4x$
$2y - 8x = 24$

17. $y = 42 - 4x$
$6x = 50 + 5y$

18. $y - 12 = 4x$
$2y = 8x + 24$

19. $2y - 2x = 7$
$y - x = 3.5$

20. Summary Explain, giving examples, and compare what happens when you try to solve the system if

a. the lines are parallel;
b. the equations represent the same line;
c. the lines meet in one point.

ADDING LINES

21. a. Graph the two lines on the same pair of axes.

$$3x + y = 7$$
$$-2x + y = -8$$

b. Label the point of intersection.
c. Add these two equations to get a third equation. Graph it on the same pair of axes. What do you notice?

22. a. Graph these two lines on the same pair of axes.

$$\text{(A)} \quad 5x - 2y = 3$$
$$\text{(B)} \quad 2x + y = 3$$

b. Label the point of intersection.
c. Get a third equation by adding.

$$\text{(A)} + \text{(B)} + \text{(B)}$$

Graph this equation on the same pair of axes. What do you notice?

23. Solve the system. $\begin{cases} 5x - 2y = 3 \\ 2x + y = 3 \end{cases}$

24. Here are two equations of lines.

$$2x + 3y = 5$$
$$x + 2y = 4$$

Use addition of these equations to get the equation of a *horizontal* line that passes through their intersection.

25. Solve the system. $\begin{cases} 2x + 3y = 5 \\ x + 2y = 4 \end{cases}$

26. Summary Explain how "adding lines" to get horizontal and vertical lines is related to solving systems of equations.

MORE MIND READING

27. Which of these problems has one solution? Which has an infinite number of solutions? Which has no solution? Explain.

a. I'm thinking of two numbers. Their sum is 10. Twice the first plus twice the second is 20.
b. I'm thinking of two numbers. Their sum is 6. Their difference is 10.
c. I'm thinking of two numbers. The second is 5 more than the first. The second minus the first is 6.

REVIEW SLOPE-INTERCEPT FORM

28. The following questions are about the graph of $y = \frac{2}{3}x - 1$.

a. Where does it meet the y-axis?

b. If you move 2 units up and 3 units to the right from the y-intercept, where are you? Is that point on the graph? Explain.

c. If you move 2 units up and 3 units to the right from the point you found in part (b), where are you? Is that point on the graph? Explain.

d. Start anywhere on the line. Move 6 units up and m units to the right, to end up on the graph. What is m? Explain.

29. Describe a fast way to graph a line whose equation is given in slope-intercept form. Use an example.

30. Write these equations in slope-intercept form.

a. $3y = 4(2 - x)$

b. $4 - 3x = y - 2$

c. $y - 4 = 3(x - 2)$

d. $\frac{y}{2} = \frac{2 - 4x}{6}$

LESSON 10.7

Using Simultaneous Equations

WRITING EQUATIONS

Some problems can be solved by solving systems of equations.

Example: The members of the advanced music class of Alaberg High School gave a spring concert. Afterwards they wanted to know how many adults had attended the concert. They knew they had sold 351 tickets, and receipts totaled \$1078.50. If adult tickets were \$4.00 and student tickets were \$2.50, how many of each kind had they sold?

Identify the variables:

Let x = the number of adult tickets.

Let y = the number of student tickets.

Write the equations:

$x + y = 351$

$4.00x + 2.50y = 1078.50$

1. Interpret the two equations in terms of this problem.
2. Solve the system. Interpret your answer.
3. The following year 536 tickets were sold, with total receipts of \$1656.50. If the ticket prices were the same, how many of each type were sold? Write and solve a system of equations.
4. Compare the system you wrote in problem 3 with the one in the example. What is the same, and what is different? Explain.

Writing and solving a system of equations is an efficient way to solve the problems in this lesson. However, there are other ways to solve them, such as using tables, graphs, or by trial and error. Regardless of what method you use, show your work clearly and express your solutions in terms of the original problem.

GADGETS AND WIDGETS

5. Ken walked into Kate's Store. "How much for five of those gidgets and eight of those gadgets?" he asked. "That would be \$11.27 without tax," Kate replied. "Oops," said Ken. "I really need eight of the gidgets and five of the gadgets." The total was \$11.87 before tax. What was the cost of a gidget? What was the cost of a gadget?
6. It takes 2.5 kg of copper and 4 kg of nickel to manufacture a widget. A smidget requires 7 kg of copper and 3 kg of nickel. How many widgets and how many smidgets could you manufacture if you had
   a. 74 kg of copper and 61 kg of nickel?
   b. 80 kg of copper and 43 kg of nickel?

MOZART

7. Liza planned to tape a 12-hour Mozart Marathon. She wanted to use a combination of 90-minute and 60-minute tapes and to fill each one completely.
   a. What possible combinations of tapes could she use?
   b. If she used a total of ten tapes, and filled all of them completely, how many of each did she use?
8. Shelly earned some money assisting with preparations for Mozart's 200th birthday party. She made \$6 per hour for addressing invitations and \$8 per hour for helping to set up the stage and auditorium for the concert. She received a total of \$352. How many hours did she work at each job?

COLLEGE APPLICATIONS

9. Garabel College newspaper reported that 1089 students had applied to the college in the two-year period 1992-1994. There were 20% more applicants in the 93-94 school year than in the 92-93 school year. How many students applied to Garabel in each of the two years?

10. The number of students applying to Garabel in 92-93 was a 12% increase over the number in 91-92. How many students applied in 91-92?

11. Ms. Pavlov, the Director of Admissions, congratulated the admissions staff. "We had 32% more applicants in 93-94 than in 91-92." What is wrong with her statement?

12. The admissions department is expanding. Their budget has been increased by \$1800 per week to hire new staff. They will hire some part-time student interviewers and tour guides at \$5.25 per hour and student secretaries at \$6.50 per hour. Interviewers and tour guides work approximately 10 hours per week, and secretaries work 15 hours per week. If they need one secretary for every five interviewers and tour guides, how many of each should they hire?

MEXICAN FOOD

At La Brea's Mexican Restaurant, you can buy a Family Feast of eight enchiladas and twelve tamales for \$19.60. The Couple's Combo has four tamales and four enchiladas, and sells for \$8.00.

13. Based on these prices, what price would you recommend for the Single's Special, which has two tamales and one enchilada?

14. Mr. G. La Brea wants to add a new item to the restaurant's menu. How should he price the Double Dozen, which has a dozen enchiladas and a dozen tamales?

CHEMISTRY

15. One solution is 80% acid. Another is 20% acid. Rosemary wants 500 liters of a solution that is 70% acid. How much of each solution should she use?

GEOMETRY

16. A 30-cm string loop goes around two thumbtacks that are ten cm apart. A third thumbtack is added, so that the loop makes a right triangle. How far is the new tack from the old ones?

17. A rectangle has perimeter 30. If you add 3 to the width, and subtract 4 from the length, the area does not change. What are the length and width of the original rectangle?

PROBLEMATIC PROBLEMS

Systems of simultaneous linear equations are an important and widely-used application of mathematics. Usually they involve many variables and are solved with the help of more advanced math, plus computers or programmable calculators. The lessons in this chapter were intended to give you an introduction to this sort of mathematics. However, some of the problems in this lesson are not very realistic.

18. **Report** Discuss:
- How could some of the problems be solved (or avoided) without using algebra?
- Which problems are backwards? (You are given information that could have been figured out only by someone who already knew the answer to the problem.)
- Which problems seem to start from unrealistic numbers?
- Which problems could arise in the real world?
- Which problems are really puzzles created to help you learn algebra?

LESSON 10.8

Lines Through Points

You will need:

- graph paper
- graphing calculator (optional)

1. **Exploration** The linear equation $y = x - 1$ has (2, 1) as a solution. Make up several more linear equations in x and y that have (2, 1) as a solution. Compare your solutions with those of other students. How many different linear equations have this solution?

FINDING COORDINATES

Hint: The problems in this section and the following one can be solved by graphing carefully.

2. A line having slope -2 passes through the point (-4, 3). Give the coordinates of three more points on the line.
3. A line having slope -3 passes through the point (5, 12). The points (a, 5) and (0, b) are on the same line. Find a and b.
4. A line passes through (2, 1) and (-2, -1). Give the coordinates of three more points on the line.
5. The points (7, -2) and (6, 2) are on a line. The points (a, 5) and (0, b) are on the same line. Find a and b.

LINES THROUGH A POINT

6. Which of the following lines pass through the point (1, -1)?
   a. $5x - 5y = 10$ b. $5x + 5y = 10$
   c. $2x - 3y = 6$ d. $-3x + 2y = 6$
7. The line $y = mx - 1$ passes through the point (3, 2). What is m?
8. The line $y = (-1/3)x + b$ passes through the point (3, 2). What is b?

FINDING THE EQUATION OF A LINE

9. Graph the line that passes through the points (1, 3) and (3, 8). Find its equation.

Ellen and Sandor wanted to find the equation of a line passing through (4, 5) and (8, -3) *without using graphing*.

10. Ellen could tell by imagining the graph that the slope of the line must be negative and the y-intercept must be greater than 5. Explain.

Ellen knew that the equation could be written in slope-intercept form as $y = mx + b$. "All I have to do is find m and b," she thought. Using the point (4, 5), she substituted values for x and y and wrote this equation in m and b,

$$5 = m(4) + b$$

which she rewrote as $5 = 4m + b$.

11. What equation in m and b did she write, using the point (8, -3)?

12. a. Find the values of m and b that satisfy both of Ellen's equations.
    b. Write the slope-intercept equation of the line passing through the points.

Sandor knew that the equation could be written in standard form as $Ax + By = C$. He substituted values for x and y and wrote two equations. One was $A(4) + B(5) = C$, which he rewrote as $4A + 5B = C$.

13. a. What was the other equation?
    b. Find some values of A, B, and C that satisfy both equations. (Many solutions are possible.)
    c. Write in standard form an equation of the line passing through the points.
    d. Compare your answer to (c) with other students' answers.
14. Show that Ellen and Sandor got equivalent answers, one in slope-intercept form and the other in standard form.
15. Find the equation of a line having slope 1.5 that passes through the point (0.5, 4).
16. Find the equation of the line through the points (2.3, 4.5) and (-6, -7). (You may round off the parameters.)
17. **Summary** Explain, with examples, your strategies for finding the equation of a line,
    a. when you know its slope and the coordinates of a point on it;
    b. when you know the coordinates of two points on it.

CELSIUS-FAHRENHEIT CONVERSION

Water freezes at 0° Celsius, which is 32° Fahrenheit. Water boils at 100° Celsius, which is 212° Fahrenheit.

18. A temperature reading can be converted from Fahrenheit to Celsius by using the formula $C = mF + b$. Find m and b by using the fact that $C = 0$ when $F = 32$, and $C = 100$ when $F = 212$.
19. Find a formula for converting Celsius to Fahrenheit.
20. What is the relationship between the formulas that you found in problems 18-19?
21. When the temperature increases by n degrees on the Celsius scale, by how much does it increase on the Fahrenheit scale? Explain.

ADDING POINTS

A line passes through the points (2, 4) and (6, 8). If you add the x-coordinates and the y-coordinates of these points you get the point (8, 12). Call this point the *sum* of the points.

22. What point is the *difference* of the points?

23. a. Find the equation of the line through (2, 4) and (6, 8).
b. Does this line also pass through the sum and the difference of (2, 4) and (6, 8)?

24. Write the equation of any line and find the coordinates of two points on the line. Find their sum and difference. Does the line pass through the sum and difference points?

25. Find the equation of a line such that the sum and the difference of any two points on the line is also on the line. To find this line, it may help to experiment with graphs. Compare your answers to problems 23-24 with other students' answers.

26. Summary What kinds of lines contain the sum and the difference of any two points on the line? Explain, giving examples and counter-examples.

27. What's wrong with this reasoning? (Hint: Think about problems 18-26.)

$$0°C = 32°F$$
$$100°C = 212°F$$
Adding equals to equals:
$$100°C = 244°F$$

DISCOVERY REAL WORD PROBLEM

28. Rearrange the letters in the sentence

I'm a pencil dot.

to create an appropriate mathematical two-word phrase. (Hint: The second word has five letters.)

THINKING WRITING

10.B Fitting a Line

You will need:

graph paper

graphing calculator (optional)

These data are about *average* 45-year-olds.

	Weight (lbs)	
Height	**Men**	**Women**
4′10″		118
5′		123
5′2″	140	129
5′4″	149	136
5′6″	158	143
5′8″	167	150
5′10″	176	158
6′	186	168
6′2″	197	
6′4″	208	

1. On the same axes, graph weight as a function of height for men and women.

The points appear to lie on two straight lines. However by looking at the differences between consecutive entries, you can see that for women, a two-inch difference in height means five more pounds between 4′10″ and 5′, while it means six more pounds between 5′ and 5′2″. This shows that the slope changes, and therefore the points are not lined up exactly.

2. Between what heights is the relationship between height and weight linear? In other words, between what heights do the points lie exactly on a line?
   a. Answer this for men and for women.
   b. Find the slope of those lines.
   c. Find the equations of the lines, in the form $W = mH + b$. (Express heights in inches.)

The equations you found can be used to predict the average weight for 45-year-old men and women *in that range*.

3. Use the equation you found to calculate the weights of a man and a woman who are each 5′5″ tall. Check that your answers are consistent with the data in the table.
4. The unit of height is the inch, the unit of weight is the pound. What is the unit and meaning of the slope in these graphs?
5. In what ranges is the slope less? Greater? Explain why, in terms of the real-world meaning of the data.

It is more difficult to find a linear function relating weight to height if you try to do it over the whole range. Finding such a function is called *fitting a line to the data.* The equation of such a line is useful as an approximate formula.

6. **Exploration** Draw a line that is close to all the data points for the men. Find its equation. (Start out by finding two points on the line you drew and use their coordinates. They do not need to be points from the table.) Do this again for the women. Compare your answers with those of other students.

7. Report Explain how you found a linear equation for these data. Your report should answer the following questions, but not be limited to them.
   - In a paragraph, summarize the information contained in the table.
   - Why is it impossible to find an exact formula relating weight and height?
   - What is the meaning of slope in this context?
   - What does your formula predict for the weight of a 5′ man? Of a 6′2″ woman? Are those predictions probably too high or too low?

DISCOVERY BEYOND SQUARE ROOTS

8. With which of the following numbers of blocks could you build a single cube with no blocks left over? If you could build a cube, give its dimensions. (You may want to use the Lab Gear or make a sketch.)

   a. 8 b. 81
   c. 216 d. 729

Say that we have:

$$64^x \cdot 64^x \cdot 64^x = 64.$$

Using the product of powers law of exponents it is easy to see what x must be:

$$64^{1/3} \cdot 64^{1/3} \cdot 64^{1/3} = 64^1.$$

9. a. What must be the value of $64^{1/3}$? (Hint: What number could you substitute for it in this equation?)
   b. Use the same reasoning to find the value of $27^{1/3}$.

   The 1/3 power of a number is called the cube root of a number. Explain why.

10. Use a law of exponents to simplify.
    a. $64^{2/3}$
    b. $8^{4/3}$
    c. $64^{1/4}$

Essential Ideas

BIKES AND TRIKES

Kathryn counted 41 wheels in the preschool yard. All of them were on bikes and trikes. (She did not count training wheels.)

1. Make a table showing some possible numbers of bikes and trikes.

2. Jana counted a total of 16 bikes and trikes in the same yard. How many of each kind were there?

LETTERS AND CARDS

Bill is on vacation and wants to write to his friends. He is going to write letters and post-cards, and wants to spend no more than \$4.75 on postage. Postcard stamps are 19 cents, and letter stamps are 29 cents.

3. a. If Bill writes only cards, how many can he write?
   b. If he writes only letters, how many can he write?
   c. If he has 20 friends and wants to write as many letters and as few postcards as possible, how many of each kind should he send?

LINEAR EQUATIONS

4. Which of these equations have the same set of (x, y) solutions as each other? Make two groups. Show your work.
   a. $2x + 3y = 0.4$　　b. $10x = 2 - 15y$
   c. $15x + 10y = 5$　　d. $x + 1.5y = 0.2$
   e. $y = -1.5x + 0.5$　　f. $3x + 2y = 1$

5. Write in standard form, $y = 6x + 7$.

6. What is the equation of a line having slope 8 that passes through $(9, 11)$?

SYSTEM SOLVING

Solve each system. Check first to see if you can tell that the system has no solution or an infinite number of solutions.

7. $\begin{cases} 6m - 4b = 0 \\ 5m + 8b = 0 \end{cases}$　　8. $\begin{cases} 4m - 3b = 2 \\ 3m + 4b = 5 \end{cases}$

9. $\begin{cases} 3a + 8b = 20 \\ 3a + b = 13 \end{cases}$　　10. $\begin{cases} 6m - 2n = 12 \\ n = 3m - 4 \end{cases}$

LEGS

11. Jeanne saw some cows and chickens. She had nothing to do, so she counted their legs and heads, over and over. Here are her results.

    The first time: 93 legs, 31 heads

    The second time: 66 legs, 16 heads

    The third time: 82 legs, 29 heads

    She counted accurately only one time. Which time was it? How many cows and how many chickens were there? Comment.

12. Jonathan saw some three-legged stools and four-legged chairs. He was bored, so he counted their legs. There were 59 legs. Then he put six pennies on each stool, and eight nickels on each chair. (He thought it would make a good math problem.)
    a. He used 118 coins. Can you tell how many chairs and stools there were? Explain.
    b. The total value of the coins was \$3.74. Can you tell how many chairs and stools there were? Explain.
    c. How many of each kind of coin did he use?

GOING NUTS

The G. Ale Bar Company also sells nuts. Cashews are \$4.95 a pound, and peanuts are \$1.95 a pound.

13. Ginger was asked to create a mix of cashews and peanuts that would cost \$2.95 a pound. What percent of the mix should be peanuts and what percent should be cashews?

CREATING SYSTEMS OF EQUATIONS

14. Create a system of equations that has the solution $x = 2, y = 7$. Compare your answer with other students' answers.

15. Create two different systems of equations that have the solution $x = 4, y = -1$. Compare answers.

16. Explain your strategy for making up a system of equations having a given (x, y) solution.

17. Make up a word problem having two variables. The problem should have a unique solution. You might use one of the following themes: different-sized bottles or cans, alien creatures having different numbers of eyes or arms. Or choose anything else you want. Be creative, but make sure the math works out.

EQUATIONS AND GRAPHS

18. The graphs of $y = 2x + 3$ and $y = -4x - 5$ meet at a point having x-coordinate $-4/3$. Solve the system.

$$\begin{cases} y = 2x + 3 \\ y = -4x - 5 \end{cases}$$

19. One of (2.5, 0.5) and (0.5, 2.5) is the solution to the system $\begin{cases} 6x + 2y = 8 \\ 9x - y = 2 \end{cases}$

Where do the graphs of $6x + 2y = 8$ and $9x - y = 2$ intersect?

POINTS ON A LINE

Susan connected (6, 0) to (2, 10) with a rubber band on her geoboard. (5, 3) and (4, 5) appeared to be on the line she formed. She wondered whether they really were.

20. Find the equation of the line through (6, 0) and (2, 10). Use algebra to check whether (5, 3) and (4, 5) are on it.

21. Mark thought the question could be answered without finding the equation of a line, by using the slope of the line connecting one point to another. Use his method and explain it.

A HEIGHT-WEIGHT FORMULA?

Many people do not like to reveal their weight, but most people don't mind telling their height. Lewis thought it would be useful to have a formula giving weight as a function of height. Lewis is 5 feet 6 inches tall and weighs 141 pounds. He made up a formula that relates his weight (in pounds) to his height (in inches).

$$W = 2(H) + 9$$

22. Verify that this formula works for Lewis's height and weight.

23. Lewis's friend Doug weighs 162 pounds and is 6 feet 1 inch tall. Does Lewis's formula work for Doug? Explain.

24. Find a formula that works for both Lewis and Doug.

25. Find two people who will tell you their height and weight. Find a formula that relates their weights to their heights.

26. Check whether the formula you found in problem 25 works to predict your weight from your height. Comment.

CHAPTER 11

A spiral staircase

Coming in this chapter:

Exploration On graph paper, you want to go from (0, 0) to (p, q), where p and q are positive whole numbers. If you travel only up and to the right, following graph paper lines, how many ways are there to do it? If you travel in a straight line, how many graph paper squares do you cross?