Numbers and Reasoning

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I enjoyed George Roy Allen, and Thacker (2020) article, "Linking Factors and Multiples to Algebraic Reasoning." The authors described how an engaging problem, combined with savvy teacher moves, can help students engage in mathematical thinking. I have used this very problem, with the same intentions, for at least 30 years and included it in *Algebra: Themes, Tools, Concepts* (1994), a book I coauthored with Anita Wah (also see the online supplemental file).

One way to extend this problem's impact is to generalize. What if you could purchase either 5 or 7 items? 5 or 10? Discussion and experimentation lead to the idea that relatively prime numbers will allow us to purchase any number past a certain point.

A visual representation allows us to rapidly find the last impossible number. For 5 and 7, arrange numbers in five columns,

1 2 3 4 5

6 7 8 9 10...

If we can get a number, we also have all the numbers beneath it (they can be reached by adding 5 repeatedly)—initially all the numbers beneath the 5 and the 7. Moreover, we can get 14, 21, and 28, plus all the numbers beneath them. We now see that 23 is the largest impossible number. But notice that 28 is 4 × 7 (the last multiple of 7 before 5 × 7.) Using p and q instead of 5 and 7, with $p < q$, we see that the number directly beneath the last impossible number is $(p - 1)q$, and thus, the last impossible number is $(p - 1)q - p = pq - (p + q)$: the product minus the sum.

REFERENCES

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Wah, Anita, and Henri Picciotto. 1994. *Algebra: Themes, Tools, Concepts*. Mountain View, CA: Creative Publications.

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