LESSON 3.12

Similar Figures

You will need:

geoboards

dot paper

EQUIVALENT FRACTIONS

1. Using a rubber band, connect the origin and (6, 9). The line misses most geoboard pegs, but it goes *exactly* over two of them (in addition to the pegs it connects). What are their coordinates?

Problem 1 provides a way to find equivalent fractions on the geoboard. If you think of (6, 9) as representing 6/9, you have found two other fractions equivalent to it, making this a set of three equivalent geoboard fractions.

2. **Exploration** Find as many sets of equivalent geoboard fractions as possible. Do not use zero in the numerator or denominator. There are 56 fractions distributed in 19 sets. Do not include sets that consist of just one fraction.

ENLARGING WITHOUT DISTORTION

3. a. Make the face of an alien with rubber bands on your geoboard. The whole face needs to fit in the bottom left quarter of the board. In other words, none of the coordinates can be greater than 5. Don't make it too complicated.

b. Make a record of the coordinates you used. You will need those in the next problems.

c. Copy the face on dot paper.

4. Doubling the x-coordinates and leaving the y-coordinates the same, make a copy of your alien's face on dot paper. This is called the $(2x, y)$ copy.

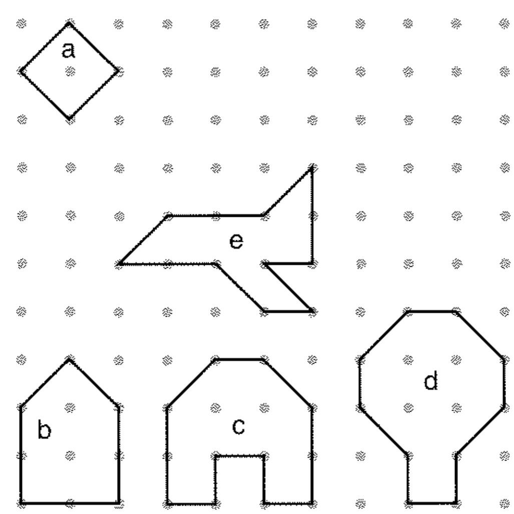
5. Repeat problem 4, but this time leave the x-coordinates as in the original and double the y-coordinates only. This is called the $(x, 2y)$ copy.

6. Repeat problem 4 again, with both x- and y-coordinates doubled. This is called the $(2x, 2y)$ copy.

7. **Summary** Write a paragraph answering these questions: Which of the copies looks most like an enlarged version of the original? How are the other copies distorted?

8. Write a story about the alien's adventures, explaining why its face went through these changes.

9. Enlarge the following figures without distortion. Explain how you did it.



SIMILAR RECTANGLES

Definition: When one figure can be obtained from another by enlarging it or shrinking it without distortion, the figures are said to be *similar*.

10. Make a rectangle having vertices at (0, 0), (4, 0), (4, 6), and (0, 6). Find a smaller rectangle that is similar to it by finding a number you can multiply the given coordinates by to get whole number coordinates that will fit on the geoboard. Sketch both on the same figure.

11. Repeat problem 10, but find a larger rectangle that is similar to the given one. Sketch it on the same figure as in problem 10.

The following questions are about the three rectangles from problems 10 and 11.

12. Connect the origin with the opposite vertex in the largest rectangle. Does your rubber band pass through vertices of the other two rectangles?

13. What are the length and width of each rectangle? How are they related to each other?

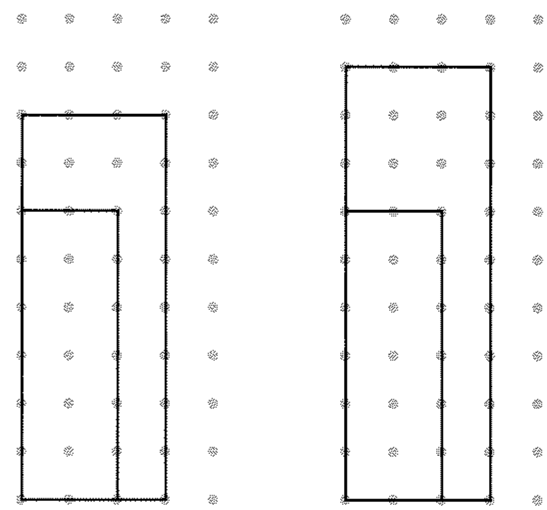
14. Can you think of a *single number* that tells what all three rectangles have in common?

Here are two ways to tell whether two rectangles are similar.

Geoboard diagonal method: Make both rectangles in the bottom left of a geoboard, with one vertex on the origin, and sides along the x- and y-axes. Then connect the origin to the opposite vertex of the larger rectangle. If the diagonal you created passes exactly over the vertex of the smaller rectangle, they are similar.

Calculator division method: Check whether the ratio of the dimensions is the same in both the rectangles.

Example:
a. a 2-by-6 rectangle and a 3-by-8 rectangle
b. a 2-by-6 rectangle and a 3-by-9 rectangle



2/6 = 0.3333333...
3/8 = 0.375
3/9 = 0.3333333...

15. Explain the results of the two methods in this example.

You may know other methods for recognizing whether fractions are equivalent. You can use those also, to determine whether rectangles are similar.

16. Summary Explain how the ideas of *similar rectangles* and *equivalent fractions* are related.

17. Are these two rectangles similar? The first one has vertices: (0, 1), (2, 0), (4, 4), and (2, 5). The other one has vertices (7, 3), (9, 6), (3, 10), and (1, 7). Since the methods outlined above will probably not work, explain how you arrive at your answer.

REVIEW ***THE COMMUTATIVE AND ASSOCIATIVE LAWS***

18. Write an expression using
- the numbers 1, 2, and -3, in any order,
- two subtractions,

in as many ways as possible.

In each case, calculate the value of the expression.

Examples: $2 - 1 - -3 = 4$
$2 - (1 - -3) = -2$
$(-3 - 1) - 2 = -6$

19. Do the commutative and associative laws apply to subtraction? Explain.

DISCOVERY ***CLOCKMATH***

Clock Math can be defined by saying that only the numbers on the face of a clock (1, 2, ..., 12) are used. In Clock Math, $5 + 9 = 2$, and $5 \times 9 = 9$. This is because when you pass 12, you keep counting around the clock.

20. Report Write a report on Clock Math. You may start with a science fiction or fantasy story to explain an imaginary origin for Clock Math. Your report should include, but not be limited to, answers to the following questions: Is there a *Clock Zero*? What is it? Does every number have a *Clock Opposite*? What is it? Is there a *Clock One*? Does every number have a *Clock Reciprocal*? What is it? Don't forget to make addition and multiplication tables.