LESSON 5.5

Graphing Parabolas

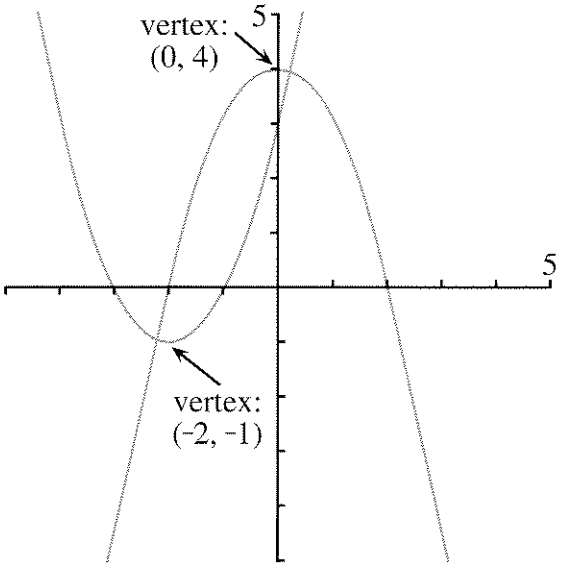
You will need:

graph paper

Definitions:

- Second-degree polynomial functions are also called *quadratic* functions.
- Graphs of quadratic functions have a special shape called a *parabola.*
- The lowest or highest point on a parabola is called its *vertex.*

Here are two quadratic functions and their graphs. Each one has two x-intercepts and one vertex.



1. What is the y-coordinate of the x-intercepts? What is the x-coordinate of the y-intercept?

2. For each parabola in the figure,
   a. what are the x- and y-intercepts?
   b. which x-intercept is the vertex closer to?

FINDING INTERCEPTS AND THE VERTEX

3. a. Copy and complete the table of values for the quadratic function $y = x^2 + 2x - 8$. Use at least six values from -5 to 5. Using the format shown will help you avoid making mistakes in computation.

x	$x^2 + 2x - 8$	y
-5	$(-5)^2 + 2(-5) - 8$	7
-4	_____	—
...	_____	—
4	_____	—
5	_____	—

   b. Use your table to make a graph of the function.
   c. Label the intercepts and the vertex.

4. Repeat problem 3 for the function $y = (x + 4)(x - 2)$.

5.
   a. Compare your graphs in problems 3 and 4. Explain what you observe.
   b. How are the x-intercepts related to the expression $(x + 4)(x - 2)$?

6. The quadratic function $y = x^2 - 6x + 8$ can be written in factored form as $y = (x - 4)(x - 2)$.
   a. Make a table of values for this function, including the intercepts and the vertex.
   b. Graph the function. Label the intercepts and the vertex.
   c. How are the x-intercepts related to the expression $(x - 4)(x - 2)$?
   d. How is the y-intercept related to the expression $x^2 - 6x + 8$?

For each problem, 7-10:

a. Write the function in factored form.

b. Make a table of values, including the intercepts and the vertex.

c. Graph the function, labeling the intercepts and the vertex.

7. $y = x^2 - 2x - 3$

8. $y = x^2 + 4x + 3$

9. $y = x^2 - 4x + 3$

10. $y = x^2 + 2x - 3$

11. ✒ Write the equation of a quadratic function whose graph would cross the x-axis at (2, 0) and (-3, 0). Explain how you know it will work.

12. ✒ Write the equation of a parabola having y-intercept -4. Explain how you know it will work.

13. **Generalization** Consider functions of the form $y = x^2 + bx + c$ that can be factored into $y = (x - p)(x - q)$.

a. How are b, c, p, and q related?

b. How would you find the coordinates of the intercepts?

c. 💡 How would you find the coordinates of the vertex?

SMILES AND FROWNS

14. Make a table of values for the quadratic function $y = (x - 4)(x - 1)$ and graph it.

15. Repeat for $y = -(x - 4)(x - 1)$.

16. Compare your graphs from problems 14-15. What is alike about the graphs and what is different? How do their x-intercepts and vertices compare?

17. Write an equation of a quadratic function whose graph satisfies these given conditions.

a. a *smile* parabola having x-intercepts (3, 0) and (-2, 0)

b. a *frown* parabola having x-intercepts (3, 0) and (-2, 0)

c. a *smile* parabola having x-intercepts (-3, 0) and (-2, 0)

d. a *frown* parabola having x-intercepts (-3, 0) and (-2, 0)

18. ✒ Explain how you know that your answers to problem 17 are correct. You may check your answers by making a table of values, and graphing.

19. ✒ Write the equation of a quadratic function that passes through the origin and (5, 0). Explain.

20. Write an equation of a quadratic function whose graph satisfies the given conditions.

a. a parabola having one x-intercept at (1, 0) and the vertex with x-coordinate 2

b. a parabola having one x-intercept at (1, 0) and the vertex at (2, 1)

c. 💡 a parabola having one x-intercept at (1, 0) and the vertex at (2, 2)

HOW MANY x-INTERCEPTS?

21. Graph each of these four quadratic functions on the same axes.

a. $y = x^2 + 6x + 5$

b. $y = x^2 + 6x + 8$

c. $y = x^2 + 6x + 9$

d. $y = x^2 + 6x + 12$

22. Write a paragraph describing and comparing the graphs you drew in problem 21. Which graph or graphs have two x-intercepts? Which have one? Which have none? Could you have predicted this before graphing? Explain.

23. Consider the quadratic function $y = x^2 + 4x +$ ___. Fill in the blank with a number that will give a function whose graph is

a. a parabola having one x-intercept;

b. a parabola having two x-intercepts;

c. a parabola having no x-intercepts.

24. Check your answers to problem 23 by graphing, or explain why you are sure you are correct.

PUZZLES *MAKING CHANGE*

25. Find the largest number of pennies, nickels, and dimes that you can have and still not be able to make change for a quarter. Explain your answer.

26. Find the largest number of coins you can have and still not be able to make change for a dollar. (Assume that you can have any coins except a silver dollar.) Explain this answer.

PREVIEW *ZEROING IN*

27. If $ab = 0$, $bc = 0$, and $ac = 1$, what is b?

28. If $abc = 0$ and $bcd = 1$, what conclusion can you draw? Explain.

PUZZLE *SQUARE SUMS*

29. Arrange the whole numbers from 1 to 18 into nine pairs, so that the sum of the numbers in each pair is a perfect square.