LESSON 5.10

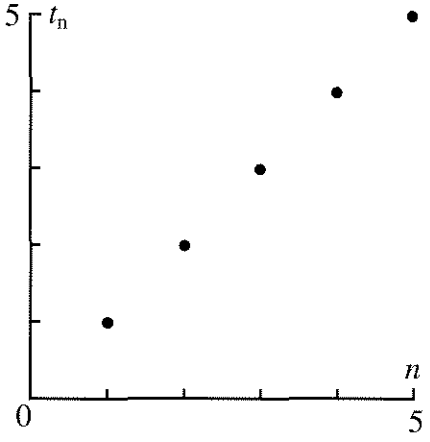
Sequences

Definitions: A *sequence* is an ordered list of numbers, called *terms*. (Notice that this is a new use of the word *term*.) Terms are often indicated with subscripted variables, such as t_1, t_2, or t_n.

GRAPHS OF SEQUENCES

Definition: The *natural numbers* are the numbers we count with: 1, 2, 3, 4, ...

The natural numbers are the easiest sequence of numbers to write using a variable. The first natural number is 1, the second natural number is 2, and so on; $t_1 = 1$, $t_2 = 2$, ... The n^{th} natural number is n, so $t_n = n$. The graph shows the sequence of natural numbers.



1. Graph the first few terms of the sequence below. Does it make sense to connect the dots? Explain.

n	1	2	3	4	...	n
t_n	2	5	8	11	...	$3n - 1$

2. Make a table, and graph the first few terms of the sequence whose n^{th} term is $t_n = 3n + 1$. Compare your graph with the one you drew in problem 1. How are they the same? How are they different?

3. You may remember this sequence.

n	1	2	3	4	...	n
t_n	1	3	6	10	...	?

a. What is the 6th term?

b. What is the n^{th} term?

c. Graph the first few terms. Is your graph a straight line?

GETTING EVEN

4. If 2 is the first even number, 4 the second, and so on, what is the millionth even number? In terms of n, what is the n^{th} even number?

5. Graph the first few terms of the sequence of even numbers. Is your graph a straight line?

n	1	2	3	...	?	n
t_n	2	6	?	...	42	?

6. The n^{th} term in the above sequence is the *sum of the first n even numbers.*

a. What is t_5?

b. Which term has a value of 42?

c. Graph the first few terms. Is your graph a straight line?

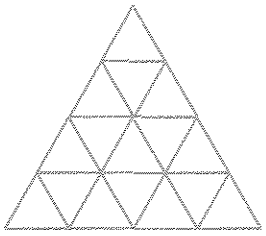
d. In terms of n, what is the n^{th} term of this sequence?

THAT'S ODD!

7. If 1 is the first odd number, 3 the second, 5 the third, what is the one-hundredth odd number?

8. a. In terms of n, what is the n^{th} odd number?

b. Graph the first few terms in the sequence of odd numbers.



9. a. Look at the figure. How many unit triangles are in the first row? The second? The third? (Count triangles whether they point up or down.)

b. If the triangle were extended indefinitely, how many unit triangles would there be in the n^{th} row?

10. a. How many unit triangles are there altogether in the first two rows? The first three rows?

b. How many unit triangles are in the first n rows?

11. What is the sum of the first two odd numbers? The first three?

12. a. What is the sum of the first n odd numbers?

b. Graph the first few terms in the sequence of sums of odd numbers.

ARITHMETIC SEQUENCES

Definition: In an *arithmetic sequence,* the difference between consecutive terms is always the same. It is called the *common difference.*

Examples: These are arithmetic sequences.
2, 7, 12, 17, 22 (The common difference is 5.)
5, 8, 11, 14, 17, 20, 23, 26, 29, 32 (The common difference is 3.)

These are not arithmetic sequences.
3, 9, 27, 81
1, -1, 1, -1, 1, -1
4, 9, 16, 25, 49

13. Which of these are arithmetic sequences? For those that are, what is the common difference?

a. 2, 6, 8, 12, 16, 20

b. 3, 6, 3, 7, 3, 8

c. 19, 13, 7, 1, -5, ...

d. the sequence of even numbers

e. the sequence of odd numbers

f. $2, 2 + 9, 2 + 2 \cdot 9, 2 + 3 \cdot 9, 2 + 4 \cdot 9$

14. Make up an arithmetic sequence for another student.

15. Answer these questions about a classmate's sequence.

a. Is it really an arithmetic sequence?

b. What is the common difference?

c. In terms of n, what is the n^{th} term?

16. For each arithmetic sequence, find the common difference, and write the n^{th} term in terms of n.

a. 2, 7, 12, 22, ...

b. $2 + 1 \cdot 5, 2 + 2 \cdot 5, 2 + 3 \cdot 5, ...$

c. $2, 2 + 1 \cdot 5, 2 + 2 \cdot 5, 2 + 3 \cdot 5, ...$

17. Answer the same questions as in problem 15 for:

a. $y, y + 1 \cdot 5, y + 2 \cdot 5, y + 3 \cdot 5, ...$

b. $2 + 1 \cdot x, 2 + 2 \cdot x, 2 + 3 \cdot x, ...$

c. $y + 1 \cdot x, y + 2 \cdot x, y + 3 \cdot x, ...$

d. $y, y + 1 \cdot x, y + 2 \cdot x, y + 3 \cdot x, ...$

18. Summary Explain how to calculate the n^{th} term of an arithmetic sequence, if you know the first term and the common difference. Test your method on several arithmetic sequences.

PREVIEW EQUATIONS

19. For each equation, find values of x_1, x_2, and x_3, that make it true.

a. $(x_1 + x_2 + x_3)/3 = 100$

b. $(x_1 + x_2 + x_3)/3 = 50$

c. $(x_1 + x_2 + x_3)/3 = 20$

d. $(x_1 + x_2 + x_3)/3 = 10$

20. For each equation in problem 19, find another set of values for x_1, x_2, and x_3 that will work.

21. If possible, find a value of x_3 to satisfy each equation.

a. $(15 + 20 + x_3)/3 = 100$

b. $(15 + 20 + x_3)/3 = 50$

c. $(15 + 20 + x_3)/3 = 20$

d. $(15 + 20 + x_3)/3 = 10$

PREVIEW ANOTHER ODD TRIANGLE

1
3 5
7 9 11
13 15 17 19
21 23 25 27 29

22. Look at the array of numbers above.

a. Write the next two rows.

b. Describe how the array is made.

23. a. Look at the middle number in rows that have a middle number. What is the pattern?

b. In rows that do not have a middle number, think of the number between the middle two numbers. What is the pattern?

c. Find the sum of the numbers in each row. What is the pattern?

24. 💡

a. What is the first number in the n^{th} row?

b. What is the last number in the n^{th} row?

c. What is the sum of all the numbers in the first n rows?