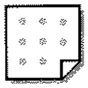
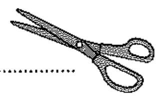
LESSON 9.9

Radical Expressions

You will need:

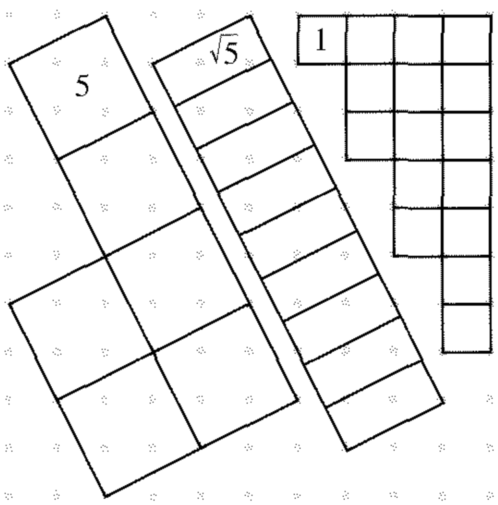
dot paper 

scissors 

MAKING RADICAL GEAR

This figure shows how to make *radical gear* from dot paper, to help model multiplications like

$$2\sqrt{5} \cdot (\sqrt{5} + 2).$$



Draw some radical gear on dot paper. Cut it out, then use it in the corner piece to do these multiplications.

1. Multiply.
   a. $2\sqrt{5} \cdot (\sqrt{5} + 2)$
   b. $\sqrt{5} \cdot (2\sqrt{5} + 2)$
   c. $4\sqrt{5} \cdot (\sqrt{5} - 1)$
   d. $3\sqrt{5} \cdot (2\sqrt{5} - 1)$

2. Multiply.
   a. $(2\sqrt{5} + 1) \cdot (\sqrt{5} + 2)$
   b. $(2 + \sqrt{5}) \cdot (\sqrt{5} + 2)$
   c. $(2\sqrt{5}) \cdot (2\sqrt{5})$
   d. $(2\sqrt{5})(2 + \sqrt{5})$

3. Multiply.
   a. $(2\sqrt{5} + 1) \cdot (2\sqrt{5} - 1)$
   b. $(\sqrt{5} + 1) \cdot (\sqrt{5} - 1)$
   c. $(3\sqrt{5} - 1) \cdot (\sqrt{5} + 1)$
   d. $3 + \sqrt{5}(2\sqrt{5} - 1)$

APPLYING THE DISTRIBUTIVE LAW

Rule: As you probably noticed, when multiplying radical expressions, *the radicals are handled as if they were variables*.

Example: You can set up a table to multiply $(\sqrt{3} - 2)(\sqrt{2} - \sqrt{3})$.

	$\sqrt{3}$	-2
$\sqrt{2}$	$\sqrt{6}$	$-2\sqrt{2}$
$-\sqrt{3}$	-3	$2\sqrt{3}$

So the product is $\sqrt{6} - 2\sqrt{2} + 2\sqrt{3} - 3$.

4. Multiply.
   a. $7\sqrt{3} \cdot (\sqrt{6} - \sqrt{3})$
   b. $(7 + \sqrt{3}) \cdot (\sqrt{6} - \sqrt{3})$
   c. $7 + \sqrt{3} \cdot (\sqrt{6} - \sqrt{3})$
   d. $(8 - 2\sqrt{3}) \cdot (\sqrt{3} + 4)$

5. Find the missing terms.
   a. $(1 + \sqrt{3})$____ $= 3 + \sqrt{3}$
   b. $\sqrt{5} \cdot$ ____ $= 10 + 4\sqrt{5}$
   c. $(6 + \sqrt{7})($___ $+ \sqrt{7}) = 55 + 14\sqrt{7}$
   d. $(\sqrt{6} + \sqrt{2}) \cdot$ ____ $= 2\sqrt{3} + 2$
   e. $(\sqrt{15} - \sqrt{2}) \cdot$ ____ $= 5\sqrt{3} - \sqrt{10}$

DISAPPEARING RADICALS

6. Find the product. Simplify your answer.
   a. $(x - y)(x + y)$
   b. $(x - \sqrt{5})(x + \sqrt{5})$
   c. $(\sqrt{3} - x)(\sqrt{3} + x)$
   d. $(\sqrt{3} - \sqrt{5})(\sqrt{3} + \sqrt{5})$

7. Explain why there are no radicals in the simplified form of any of the answers to problem 6.

8. For each binomial, find a binomial to multiply it by so that the result has no radicals.
   a. $(\sqrt{7} - \sqrt{8})$ b. $(\sqrt{x} + \sqrt{y})$
   c. $(2 - \sqrt{y})$

FRACTIONS AND RADICALS

Definition: To *rationalize* the denominator (or numerator) of a fraction is to write an equivalent fraction with *no radicals* in the denominator (or numerator).

9. Rationalize the denominator. $\frac{1}{2 + \sqrt{3}}$

10. In problem 9, Gerald tried to multiply the numerator and denominator by $(2 + \sqrt{3})$. Explain why this did not work.

11. Daniel used the idea in the section **Disappearing Radicals** to rationalize the denominator. Explain what he did, and why it did work.

12. Rationalize the denominator.
    a. $\frac{1}{\sqrt{2} + 3}$
    b. $\frac{1}{3 - \sqrt{3}}$
    c. $\frac{4}{\sqrt{5} - \sqrt{6}}$
    d. $\frac{5}{\sqrt{5}}$

13. Rationalize the numerator.
    a. $\frac{7 - \sqrt{5}}{4}$ b. $\frac{\sqrt{7} - \sqrt{5}}{4}$

REVIEW CALCULATOR EXPERIMENTS

14. Use your calculator to compute
    $(\sqrt{9876} - \sqrt{9866})(\sqrt{9876} + \sqrt{9866})$.
    Comment on the answer.

15. Bernard believes that the square root of the square of a number is the number itself.
    a. Is he right or wrong? Explain.
    b. What's the square root of the square of -543? Make a prediction, then use your calculator to check.

16. Choose any number. Find its square root on your calculator. Then find the square root of the result. Continue this until you notice something happening. What is happening? Can you explain it? What starting numbers does it work for?

17. Always, sometimes, or never? Explain, using examples.
    a. $x^2 > x$ b. $1/x^2 > 1/x$
    c. $\sqrt{x} < x$ d. $1/\sqrt{x} > 1/x$
    e. $\sqrt{x} < x^2$ f. $1/\sqrt{x} > 1/\sqrt{x^2}$

REVIEW GEOBOARD PUZZLES

18. If two sides of geoboard triangle are $\sqrt{2}$ and $\sqrt{5}$, what are the possibilities for:
    a. the third side?
    b. the area?

19. Find the geoboard figure having the least area, if its perimeter is
    a. 20; b. $4\sqrt{65}$;
    c. $10 + 2\sqrt{65}$; d. $10\sqrt{2} + 2\sqrt{85}$.