LESSON 10.2

How Much of Each Kind?

AT THE LAUNDROMAT®

1. **Exploration** Some dimes and quarters have a total value of $3.95. How many of each coin might there be? (Find all the possibilities.) What is the fewest coins there could possibly be? The most? Explain, showing your method of thinking about this problem and commenting on any patterns you notice.

Dan needs nickels and quarters to do his laundry at Science and Math Quick Wash. He has a five-dollar bill. The table shows one possible combination of coins he might get if he asks for change in nickels and quarters only. (The value is given in cents.)

Nickels		Quarters		Total Coins	
no.	**value**	**no.**	**value**	**no.**	**value**
45	225	11	275	56	500

2. Add at least six more possibilities to the table and comment on any patterns you notice. (If you don't see any patterns, add more possibilities until you do.)

3. What is the fewest coins Dan might get? The most?

4. Would it be possible for Dan to have an even number of coins? An odd number? Explain.

5. Would it be possible for Dan to have the same number of quarters as nickels? If so, how many of each would he have?

If Dan gets x nickels and y quarters, the entry in the table would look like this.

Nickels		Quarters		Total Coins	
no.	**value**	**no.**	**value**	**no.**	**value**
x	$5x$	y	$25y$	—	—

6. a. Explain the meaning of the expressions $5x$ and $25y$ in the table.

b. Complete the entry, giving the total number of coins and their value *in terms of x and y.*

Any possible whole number pair of values (x, y) giving a possibility for the number of nickels and quarters that Dan might get in change will satisfy this *equation*,

$$5x + 25y = 500.$$

For example, it is easy to show by substitution that the pair (45, 11) satisfies this equation: $5(45) + 25(11) = 500$. This pair also satisfies this condition, or *constraint:*

The total number of coins is 56.

7. Is there another (x, y) pair that satisfies the same equation and the same constraint? If so, what is it?

8. Find (x, y) pairs that satisfy both the equation $5x + 25y = 500$ and the constraints given. (You may want to extend the table you made. You can save work by looking for patterns in your table.) Some may not be possible.

Constraints:

a. The total number of coins is 80.

b. There are 20 times as many nickels as quarters.

c. There are 12 more nickels than quarters.

d. There are 8 more quarters than nickels.

9. Each of the constraints in problem 8 can be expressed as an equation in x and y. Write each equation.

10. At Science and Math Quick Wash, the machines take three quarters and one nickel to wash and one quarter to dry. If Dan wants to do as many loads as possible,

a. how many loads of wash will he be able to do?

b. what change should he request for his five-dollar bill? (Find all possible answers.)

CRANBERRY-APPLE JUICE

Nelson works for the G. Ale Bar Company, a chain of soda fountains. He is trying to create a special recipe to make a best-selling juice. For a taste test, he wants to prepare several 20-cup batches of different mixtures of pure apple juice with a cranberry-apple juice that is 50% apple and 50% cranberry.

Apple juice		**Cranberry-apple**		**Mixture**	
apple	**cran**	**apple**	**cran**	**apple**	**cran**
15	0	2.5	2.5	17.5	2.5
8	0	6	6	14	6
6	0	—	—	—	—
—	—	8	—	—	—
—	—	—	9.5	—	—
x	—	$0.50y$	—	$x+0.50y$	—

11. Copy and complete the table. Add several more numerical possibilities. (The last row is based on x cups of apple juice and y cups of cranberry-apple.)

12. What are the largest and smallest amounts of cranberry juice possible in one of Nelson's mixtures? What about apple juice? Explain.

13. For the mixture in the first row, what percent *of the total* is cranberry? (Be careful! You can't get the answer by dividing 2.5 by 17.5)

14. Add a **% cran** column to your table, and repeat problem 13 for all the rows.

15. What are the largest and smallest *percentages* of cranberry juice possible in one of Nelson's mixtures? What about apple juice? Explain.

16. Explain why the expression $x + 0.50y$ represents the total amount of apple juice in the mixture.

17. What is the expression for the total amount of cranberry juice in the mixture? Explain.

18. Explain why $x + y$ equals 20 for every possibility listed in the table.

19. What does the $x + 0.50y = 10$ mean in this situation? Is there an (x, y) pair that satisfies this equation? Explain.

For each equation, 20-25:

a. Interpret the equation in terms of this situation.

b. If possible, find a value of x and of y that satisfies the equation, given the constraint that x and y add up to 20.

20. $0.50y = 4$

21. $x + 0.50y = 15$

22. $x + 0.50y = 4$

23. $x + 0.5y = 11.5$

24. $x + 0.50y = 0.75(x + y)$

25. $x + 0.50y = 0.25(x + y)$

26. Which of the equations 20-25 were impossible to solve? Would they have been possible if the total amount had been 30 cups? Explain.