LESSON 14.2

Simplifying Algebraic Fractions

You will need:

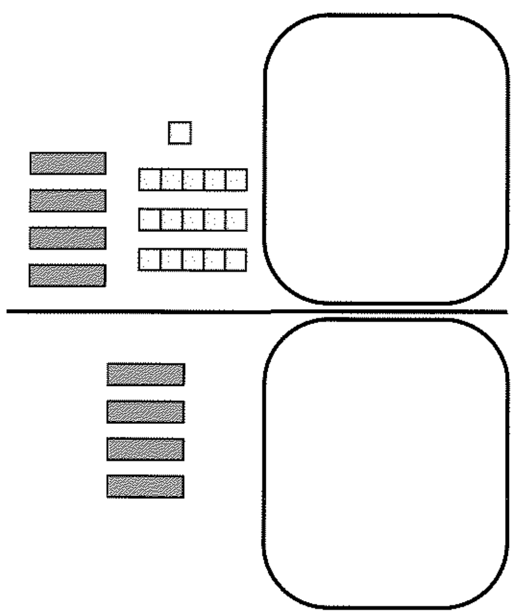
Lab Gear 

ADDING OR SUBTRACTING EQUAL AMOUNTS

1. **Exploration** What happens if you add or subtract equal amounts to or from the numerator and the denominator of a fraction? How can you tell whether the value of the fraction will increase, decrease, or remain the same? Make up several examples to see what happens, then make a generalization.

To model fractions with the Lab Gear, you can use the workmat turned on its side. Instead of representing an equals sign, the straight line in the middle now represents the fraction bar.

Edith and Anna modeled the fraction $\frac{4x + 16}{4x}$ with the Lab Gear, as shown below.



"This is an easy problem," said Edith. "There's a $4x$ in both the numerator and the denominator, so I can get rid of them. The simplified fraction is 16."

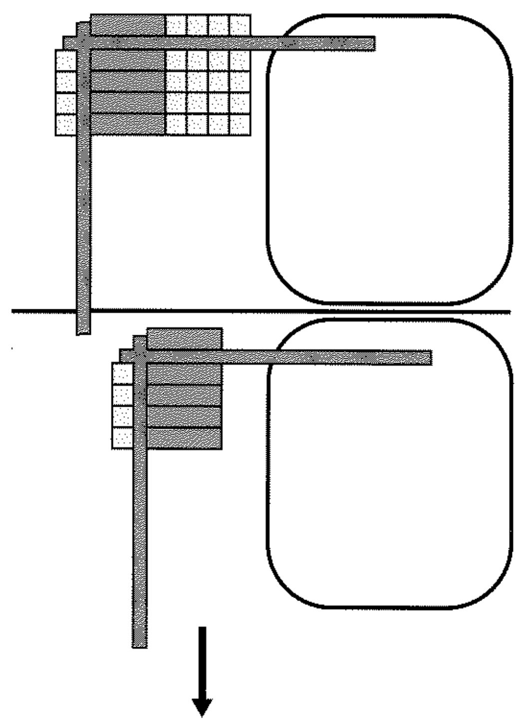
Anna didn't think Edith's method was right. She decided to check Edith's answer by substituting.

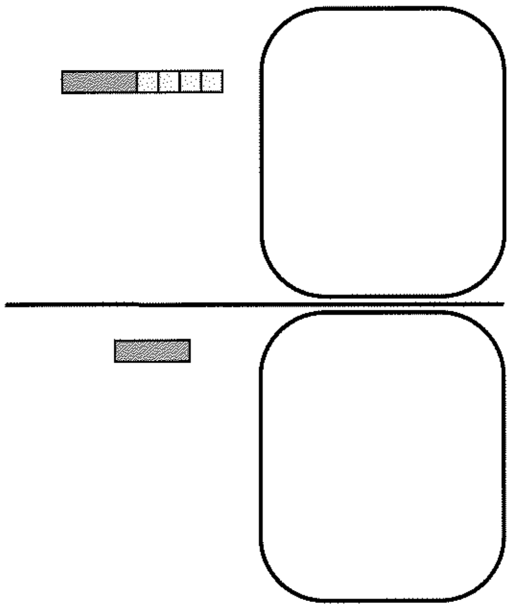
2. Calculate the value of the expression $\frac{4x + 16}{4x}$ for several different values for x. Do all values of x make this fraction equal to 16? Does any value of x make it equal to 16? Explain.

3. Explain why you cannot simplify a fraction by subtracting the same number from the numerator and the denominator. Give examples.

COMMON DIMENSIONS AND DIVISION

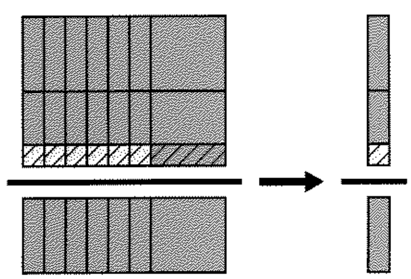
As you know, to simplify a fraction, you *divide numerator and denominator by the same number*. This is still true of algebraic fractions.





4. Study the previous figure.
   a. What are the numerator and the denominator divided by?
   b. What is the simplified fraction?

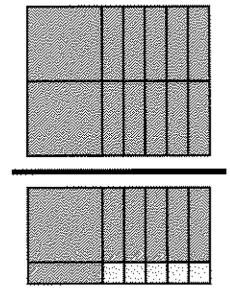
Sometimes, as in the figure below, the numerator and denominator rectangle are seen to have a common dimension, which is the common factor we divide by to get the simplified fraction.



5. Study the preceding figure.
   a. Write the original fraction.
   b. Show what the numerator and denominator must be divided by to simplify the fraction.
   c. Write the simplified fraction.

Repeat problem 5 for the following figures.

6.



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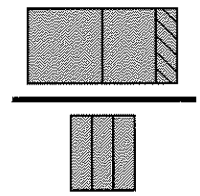
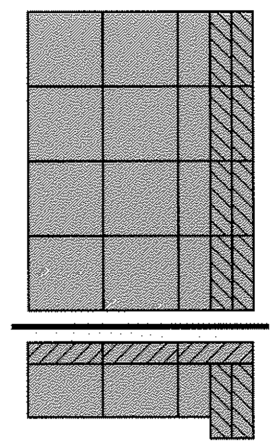


8.



SIMPLIFYING FRACTIONS

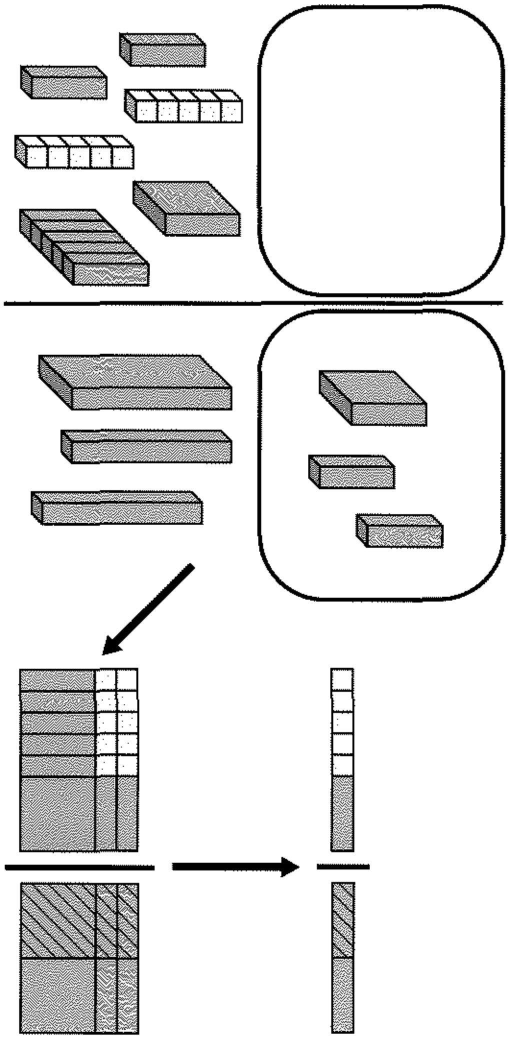
Sometimes it is necessary to factor the numerator and the denominator in order to see the common factors.

Example: Simplify: $\frac{x^2 + 3x + 2}{x^2 + 5x + 6}$

Factor: $\frac{(x + 2)(x + 1)}{(x + 2)(x + 3)}$

Divide both numerator and denominator by the common factor, $(x + 2)$. The simplified fraction is: $\frac{x + 1}{x + 3}$.

The following example is done with the Lab Gear.



9. Explain the process shown in the figure, using words and algebraic notation.

If possible, simplify these fractions.

10. $\frac{3x + 12}{x^2 + 4x}$

11. $\frac{x^2 + 10x + 25}{2x + 10}$

12. $\frac{7x + 5}{7x}$

13. $\frac{2d + 3}{d + 3}$

ZERO IN THE DENOMINATOR

When we substitute 2 for x in the fraction $\frac{3x - 1}{x - 2}$, the denominator has the value zero. Since division by 0 is undefined, we say that the fraction is undefined when $x = 2$.

For what value or values of x (if any) is each fraction undefined?

14. $\frac{2x}{x - 6}$

15. $\frac{x - 6}{x + 6}$

16. $\frac{3}{2x + 6}$

17. $\frac{x^2 + 2}{x^2 - 6x + 8}$

ALWAYS, SOMETIMES, NEVER

Since $\frac{x^2 + 12x + 20}{x + 2}$ can be written

$$\frac{(x + 10)(x + 2)}{x + 2},$$

we can write:

$$\frac{x^2 + 12x + 20}{x + 2} = x + 10$$

18. Explain why the preceding equality is not true when $x = -2$.

19. Explain why it's true when $x \neq -2$.

20. For what value(s) of x is

a. $\frac{2x - 3}{8x - 12} \neq \frac{1}{4}$?

b. $\frac{x^2 - 9}{x - 3} = x + 3$?

Tell whether each equation 21-23 is always true or only sometimes true. If it is only sometimes true, give the values of x for which it is *not* true.

21. $\frac{8x}{4} = 2x$

22. $\frac{x^2 - 1}{8x - 8} = \frac{(x + 1)}{8}$

23. $\frac{5 - 5x}{2x^2 - 2} = \frac{-5}{2x + 2}$

Tell whether each equation 24-26 is always, sometimes, or never true. If it is sometimes true, give the values of x that make it true.

24. $\frac{5x - 5}{5} = 5x$

25. $\frac{5x - 5}{5} = x - 5$

26. $\frac{x^2 - 10}{5} = x^2 - 2$