LESSON 14.4

Finding the Vertex

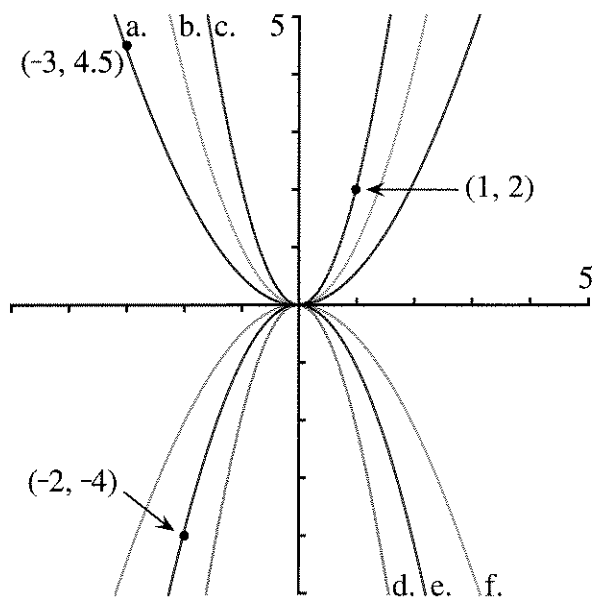
You will need:

- graph paper
- graphing calculators (optional)

Knowing more about quadratic functions and their graphs will help you understand and solve quadratic equations. In particular, it is useful to know how to find the vertex and the x-intercepts of quadratic functions in the following two forms:

- *Intercept form:* $y = a(x - p)(x - q)$
- *Standard form:* $y = ax^2 + bx + c$

DIFFERENT SHAPES

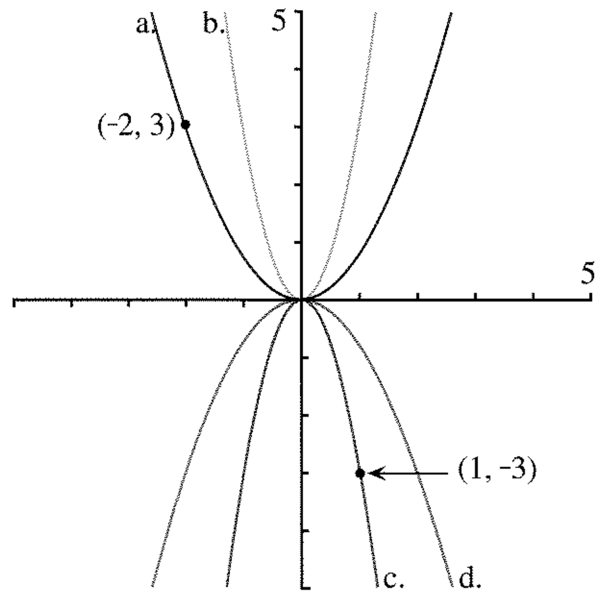


1. The figure shows several parabolas whose x-intercepts, y-intercept, and vertex are all (0, 0). Match each one with an equation:

$y = x^2$ $\quad y = 0.5x^2 \quad y = 2x^2$

$y = -x^2 \quad y = -0.5x^2 \quad y = -2x^2$

2. What is the value of a for the parabolas on the following figure?



3. Which among the parabolas in problems 1 and 2 is most open? Most closed? How is this related to the value of a?

4. Write the equation of a parabola that lies entirely between parabolas 1a and 1b.

5. Describe the graph of:
a. $y = -0.01x^2$; b. $y = 100x^2$.

6. **Summary** Explain the effect of the parameter a, in the function $y = ax^2$, on the shape and orientation of the graph.

INTERCEPT FORM

As you learned in Chapter 13, when the equation is in intercept form, you can find the vertex from the x-intercepts, which are easy to locate.

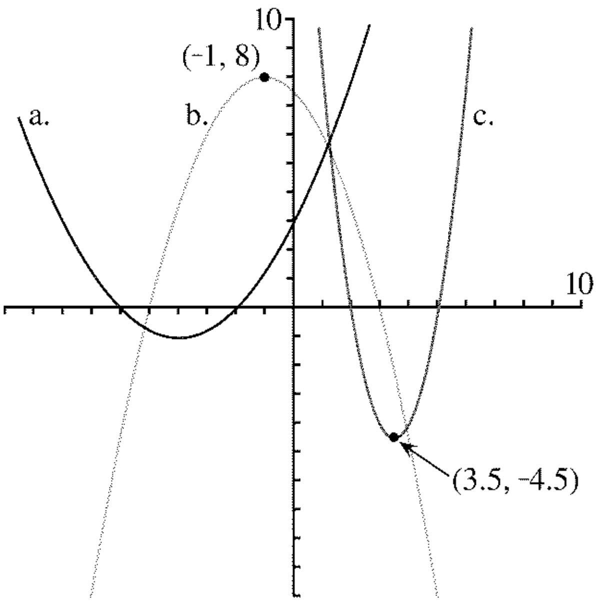
7. Try to answer the following questions about the graph of $y = 2(x - 3)(x + 4)$ without graphing.
a. What are the x- and y-intercepts?
b. What are the coordinates of the vertex?

8. **Generalization**

a. What are the x- and y-intercepts of $y = a(x - p)(x - q)$? Explain.

b. Explain in words how to find the vertex if you know the intercepts.

9. The figure shows the graphs of several parabolas. Write an equation for each one. (Hint: To find a, use either the y-intercept or the vertex and algebra or trial and error.)



10. For each equation, tell whether its graph is a smile or a frown parabola, without graphing. Explain your reasoning.

a. $y = 9(x - 8)(x - 7)$

b. $y = -9(x - 8)(x - 7)$

c. $y = 9(8 - x)(x - 7)$

d. $y = 9(8 - x)(7 - x)$

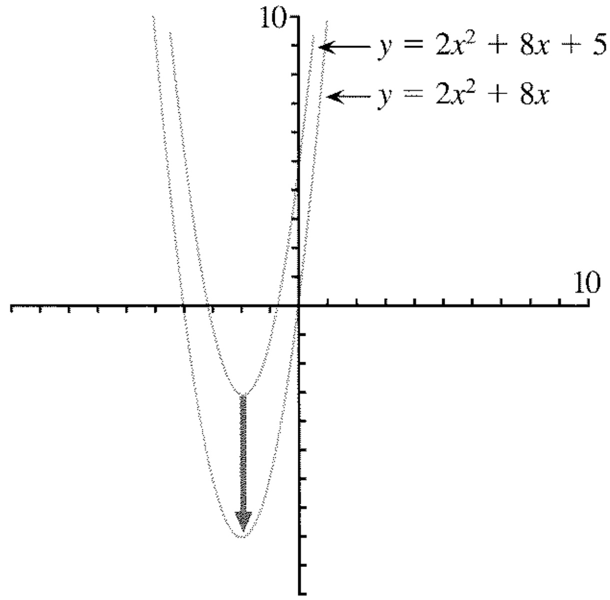
11. If you know all the intercepts and the vertex of $y = 3(x - p)(x - q)$, explain how you would find the intercepts and the vertex of $y = -3(x - p)(x - q)$.

STANDARD FORM

When the equation is in standard form, $y = ax^2 + bx + c$, it is more difficult to find the location of the vertex. One particularly easy case, however, is the case where $c = 0$.

12. Explain why when $c = 0$, the parabola goes through the origin.

13. Find the vertex of $y = 2x^2 + 8x$. (Hint: Factor to get into intercept form.)



14. How are the two graphs related? Compare the axis of symmetry and the y-intercept.

15. How is the graph of $y = 2x^2 + 8x - 3$ related to them?

16. Find the equation of any other parabola whose vertex is directly above or below the vertex of $y = 2x^2 + 8x$.

FINDING *H* AND *V*

Example: Find the coordinates (H, V) of the vertex of the graph of $y = 3x^2 - 18x + 7$.

- $y = 3x^2 - 18x$ is the vertical translation for which $V = 0$. By factoring, we see it is equal to $y = x(3x - 18)$.
- To find the x-intercepts of $y = 3x^2 - 18x$, we set $y = 0$. By the zero product property, one x-intercept is 0. To find the other, we solve the equation $3x - 18 = 0$, and get $x = 6$.
- Since the x-intercepts are 0 and 6, and the axis of symmetry for both parabolas is halfway between, it must be 3. So $H = 3$.

- Substitute 3 into the original equation to see that the y-coordinate of the vertex is:

  $$V = 3(3)^2 - 18(3) + 7 = -20.$$

  So the coordinates of the vertex for the original parabola are (3, -20).

17. For each equation, find H and V. It may help to sketch the vertical translation of the parabola for which $V = 0$.

a. $y = x^2 + 6x + 5$
b. $y = 2x^2 + 6x + 5$
c. $y = 3x^2 - 6x + 5$
d. $y = 6x^2 - 6x + 5$

Generalizations

18. What is the equation of a parabola through the origin that is a vertical translation of $y = ax^2 + bx + c$?

19. Show how to find the axis of symmetry of:

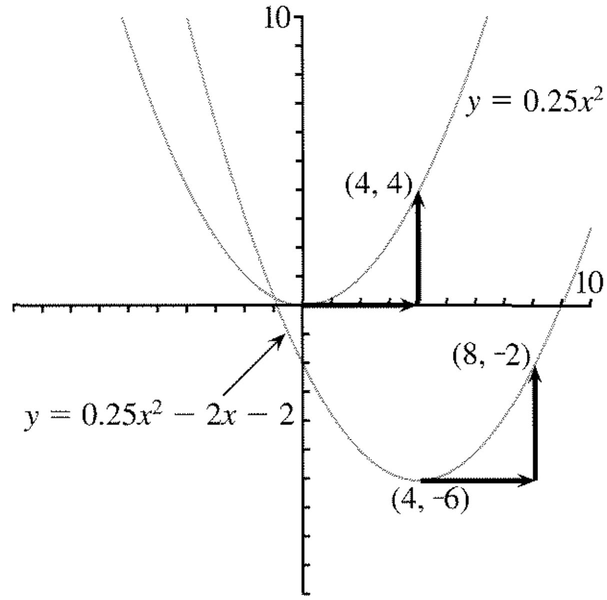
a. $y = ax^2 + bx$;
b. $y = ax^2 - bx$.

20. Explain why the x-coordinate of the vertex of the parabola having equation $y = ax^2 + bx + c$ is

$$H = -\frac{b}{2a}.$$

SAME SHAPE

The parameter a determines the shape of the parabola. The graphs of all equations in standard form that share the same value for a are translations of the graph of $y = ax^2$.



For example, the two parabolas in the figure have equations with $a = 0.25$. Therefore they have the same shape, as the following exercise shows.

21.

a. Show algebraically that starting at the vertex, and moving 4 across and 4 up, lands you on a point that satisfies the equation in both cases.
b. If you move 2 across from the vertex, show that you move up the same amount to get to the parabola in both cases.