

Kinds of Numbers

The first numbers people used were whole numbers. It took many centuries to discover more and more types of numbers. The discovery of new kinds of numbers is related to the attempt to solve more and more equations. The following equations are examples.

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|----------------|----------------|
| a. $x + 2 = 9$ | b. $x + 9 = 2$ |
| c. $2x = 6$ | d. $6x = 2$ |
| e. $x^2 = 9$ | f. $x^2 = 10$ |
| g. $x^2 = -9$ | |

1. Pretend you only know about the *natural numbers*. (These are the positive whole numbers and 0.) List the equations above that can be solved.
2. Pretend you only know about the *integers*. (These are positive and negative whole numbers, and zero.) List the equations above that can be solved. Find one that has two solutions.
3. Pretend you only know about the *rational numbers*. (These are all fractions, positive, negative, and zero. Of course, integers are included, since for example $3 = 6/2$.) List the equations above that can be solved.
4. The *real numbers* include all rational and irrational numbers. Which equations can be solved if you know about all the real numbers?

Natural numbers, integers, rational numbers, and real numbers can all be found on a one-dimensional number line. However, to solve equation (g), you need to get off the number line! The solution is a *complex number*, and it is written $3i$. The number i is a number one unit away from 0, but off the number line. It is defined as a number whose square is -1:

$$i^2 = -1.$$

Complex numbers cannot be shown on a line. They require a two-dimensional number plane.

5. Create an equation whose solution is:
 - a. a natural number
 - b. an integer, but not a whole number
 - c. a rational number, but not an integer
 - d. an irrational number
6. Create an equation that has no real number solution.