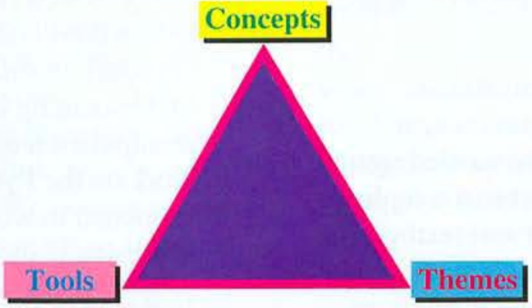
WHAT ARE THEMES, TOOLS, AND CONCEPTS?

*T*his book is organized around the interaction of **themes**, **tools**, and **concepts**.



This organizational scheme differs dramatically from the straightforward, topical organization of a traditional algebra textbook. Mathematics is a rich subject, and the interaction of these three components enables the beginning algebra student to get a glimpse of that richness. Using different tools and themes, we can revisit a concept repeatedly, with tools providing the medium for exploration, and themes the motivation. Students' understanding of a concept can deepen gradually as they work with it in new contexts and with a variety of representations.

THEMES

The tools help students learn algebra, but for most of them to want to learn it, algebra must be seen to have a connection with reality. Themes are mathematics-rich contexts, drawn from real-world or fanciful problems, where algebra concepts can be introduced, explored, developed, and reviewed. Well-chosen themes can bring algebra to life, uncover connections to other parts of mathematics, and support the claim that algebra does indeed have applications.

We selected a half-dozen themes: *dimensions*, *measurement*, *making comparisons*, *growth and change*, *optimization*, and *motion*. Some are the focus of one chapter, others appear again and again throughout the book.

▶ Making Connections

In addition to providing a context and motivation for algebraic work, the thematic approach makes it possible to reveal connections with science, culture, and daily life. Most ideas are introduced by starting with a problem outside of mathematics which leads to the realization that it is best solved with the help of algebra. For example, Chapter 3 begins with a get-rich-quick scheme designed to introduce inverse operations. Chapter 6 opens with a comparison of car rental prices which sets the stage for a series of lessons on solving equations and inequalities. In Chapter 7, identities involving squares are introduced through a problem about square windows. In Chapter 8, slope is introduced by analyzing data on children's growth rates, which allows the introduction of the laws of exponents through population growth problems. In Chapter 10, techniques for solving simultaneous equations are developed through a whole series of applications, from organizing a van pool, to making cranberry-apple juice.

Connections are made within algebra through a spiraling curriculum, where important algebraic ideas are tackled again and again. This is in contrast to the topical organization of more traditional textbooks where a topic is addressed only within the confines of its chapter. Thus, connections are made between apparently unrelated topics that turn out to have the same underlying mathematical structure. For example, in some traditional textbooks, almost everything about lines and linear functions is covered in a few sections, usually in a chapter on graphing. In this text, students encounter linear functions and their graphs repeatedly in Chapters 1 through 5.

Connections are made with other parts of mathematics throughout the book, such as exploratory lessons in number theory, with advanced algebra (through work with nonlinear functions and graphs and the graphical solution of equations and inequalities), with probability and data analysis modeling, with calculus, and especially with a full geometry strand.

▶ Integrating the Mathematics Curriculum

Merely juxtaposing traditional lessons in algebra and geometry does not mean a curriculum is integrated. In fact, it could just mean a hodgepodge of unrelated lessons. The interweaving of algebra and geometry in this text is always meaningful. The geometry topics originate in the study of perimeter, area, and volume. These topics are used to develop algebraic generalizations from numerical data, and form the basis of our approach to the distributive law and factoring. Later, they develop into lessons on similarity, which support algebraic work on proportions and slope. A geometric approach to square roots leads to a better understanding of the algebraic rules for the manipulation of radicals. And finally, the work on the Pythagorean theorem is grounded in work on distance in the Cartesian plane.

In this way, the introduction of geometric ideas is not a distraction from the teaching of algebra. In fact, it enhances it. These intimate links between algebra and geometry were at the core of ancient Greek mathematics, but have unfortunately vanished from our algebra classes.

▶ Mathematical Modeling

The final benefit of the thematic approach is that it makes it possible to start from data. Again and again we ask students to analyze data and develop mathematical models with the help of various tools: in-out tables, graphics, function diagrams, and calculators. It is this work with numbers that is the essence of the scientific enterprise, and it is in this kind of work that mathematical patterns emerge both for the professional scientist and for the algebra student.

Because this is an introductory course, we use fairly "well-behaved" data, and some of it is admittedly contrived, to fit the needs of the lessons. But we also include raw, real-world data in many lessons and occasionally encourage you and your students to collect your own data and conduct your own experiments.

Sophisticated statistical techniques are premature at this level, but we do include some work on averages and on fitting the median-median line.

With the exception of a few problems in Chapter 12, we avoid the type of applications in which a formula is supplied and students are simply given a few questions to answer by manipulating symbols. Although these applications may convince them that algebra is useful, they do little to promote students' ability to use it. We concentrate instead on mathematical modeling, in which a situation is presented and students are guided in making a mathematical analysis of it. This has several advantages for students. It immerses them in the mathematics. It involves them in the decisions that need to be made to simplify a real-world situation, in order to make a mathematical model. And it convinces them not only that mathematics is useful, but that they are capable of using it.

▶ Recreational Mathematics

As indicated in the previous paragraphs, we support the moves to make mathematics relevant. However, some reformers are getting carried away with this, and forgetting that *mathematics is worth pursuing not only because it is powerful, but also because it is beautiful and entertaining*. As the world's leading recreational mathematician, Martin Gardner, put it, "...there must be an interplay of seriousness and frivolity. The frivolity keeps the reader alert. The seriousness makes the play worthwhile."

To keep this in mind, we have included many puzzles from recreational mathematics, and many lessons are motivated not by real-world problems, but by mathematical questions that are interesting for their own sake.

Tools

The extensive use of math tools is probably the most distinguishing feature of this book. Our experience with electronic and manipulative tools has shown us that they can be the key to a student-centered classroom. They can help transform any math class into a lab course, where students experiment and make discoveries. They can help students build on their areas of competence, such as visual, electronic, or manipulative talents, and they can help provide an environment where students can improve their communication and reasoning abilities.

▶ Why Tools?

Our interest in the tool-based approach was inspired in part by the ideas of Seymour Papert. In the introduction to his book *Mindstorms,* Papert described how his early fascination with gears gave him an *object-to-think-with* while he was learning mathematics as a child in school. He then explains his belief in the importance of such objects for all children.

What an individual can learn, and how he learns it, depends on what models he has available....The gear can be used to illustrate many powerful "advanced" mathematical ideas, such as groups or relative motion. But it does more than this. As well as connecting with the formal knowledge of mathematics, it also connects with the "body knowledge"...of a child....It is this double relationship—both abstract and sensory—that gives the gear the power to carry powerful mathematics into the mind.

We must ask why some learning takes place so early and spontaneously while some is delayed many years or does not happen at all without...formal instructions....If we really look at the "child as builder" we are on our way to an answer. All builders need materials to build with....In some cases the *culture supplies them in abundance, thus facilitating constructive Piagetian learning. But in many cases where Piaget would explain the slower development of a particular concept by its great complexity or formality, I see the critical factor as the relative poverty of the culture in those materials that would make the concept simple and concrete.*

Papert's proposals are centered around the computer, but as he illustrates in his story about gears, *objects-to-think-with* need not be electronic. Our definition of tools for learning mathematics includes manipulative and paper-and-pencil tools, as well as electronic ones. All of them are *objects-to-discuss* and *objects-to-write-about*, as well as *objects-to-think-with.* Using a wide variety of tools makes algebra accessible to students with a wide variety of learning styles.

▶ Which Tools?

In order to give students a wide range of algebraic experiences, we use eleven different tools. Some of them are familiar to all math teachers, others may be new to you. Some tools are used throughout the course, others only in a few lessons. In all cases, we have found the tools to be helpful in getting students involved in, and thinking about, the major concepts of algebra.

The tools can be roughly divided into three main groups: **manipulatives** (geoboards, the Lab Gear, *radical gear)*, **pencil-and-paper tools** (grid paper, dot paper, Cartesian graphing, function diagrams, tables of values, symbol manipulation), and **electronic tools** (calculators, and, optionally, graphing calculators). A calculator, ruler, and paper and pencil are assumed to be available at all times. If manipulatives, special paper, or a graphing calculator are needed for a lesson, it is noted at the beginning.

▶ How Are Tools Used?

■ Cartesian Graphing

The Cartesian graph has had an enormous influence on how we teach, and even how we understand, mathematics. (For example, think about the use of the word *slope* to describe the rate of change of a function.) Most of us recognize it as a profound and powerful tool for visualizing algebra.

In the traditional Algebra 1 course, however, graphing is usually taught as an end in itself, just another topic to be studied briefly before moving on to something else. In this course, Cartesian graphing is introduced at the very beginning and used as a tool to analyze and visualize almost every important concept in the course, from operations with signed numbers to the quadratic formula.

Early in the course, lessons are designed to give students a thorough understanding of the relationship between three representations of functions: equation, graph, and number pairs. Throughout the book, we emphasize the application of graphing in an applied setting, giving students plenty of practice choosing appropriate scales for the axes and working with different units of measurement.

■ Graphing Calculators and Computers

The advent of graphing calculators and graphing software has made it possible to revolutionize the teaching of functions. The ability to graph any function rapidly and accurately in any domain where it is defined, and the increasing availability of the tools to do it, means that traditional paper-and-pencil graphing, as a skill, no longer deserves the central place it once occupied in our curriculum.

We are enthusiastic proponents of electronic graphing and have been involved in it for many years. In fact, a Logo grapher programmed by one of us (part of *Logo Math: Tools and Games,* published by Terrapin, Inc.) was used to generate many of the graphs for this book. Having been intimately involved with this new technology for several years has given us a good sense of both its strengths and limitations.

This book contains a number of lessons which would be enhanced by the use of graphing calculators. However, since many teachers still do not have access to them, we have designed the book so that it does not require their use.

While graphing by hand is no longer an essential skill, it can be a crucial step in the learning process If it is accompanied by discussion and reflection, paper-pencil graphing can help reveal fundamental ideas, such as the relationship between the fact that a number pair satisfies an equation, and that the point that represents it is on the graph of the function. Using a graphing calculator well involves making some judicious choices about when and where to introduce it.

■ Other Function Tools

Function Diagrams: One thing that makes Cartesian graphs powerful, but also difficult for beginners to understand, is the fact that the two axes are perpendicular. Function diagrams are another representation of functions which is based on parallel x- and y-number lines. Function diagrams complement Cartesian graphing by emphasizing different features of functions. For example, domain and range, the definition of function, inverse function, and rate of change, are easier to understand with the help of this tool than with Cartesian graphs. In fact, function diagrams make it possible to talk about many features of functions that were previously postponed until Algebra 2 or pre-calculus.

In-Out Tables: The function diagram is actually a graphic representation of a table of x- and y-values. The table of values is an essential tool from the toolbox of the traditional algebra course. It requires no special equipment. It helps students learn how to detect patterns in numerical data. A strong understanding of functions is impossible without facility in using this representation.

■ The Lab Gear®

The tools we have discussed thus far offer different representations of the concept of function. However, algebra is far more than the study of functions, and the beginning algebra student also needs tools for understanding the crucial concepts of variables, operations, equations, and inequalities. The Lab Gear is a powerful tool for studying these ideas, providing a comprehensive algebra manipulative environment.

The Lab Gear has been designed to be used for learning and understanding the distributive law, factoring, equation solving, completing the square, and many other topics of beginning algebra. Most importantly, it helps improve the discourse about symbols, by providing something concrete to manipulate and talk about. As with any of the tools, you will need to learn it yourself before trying to teach with it.

Because algebra is an extension of arithmetic, the Lab Gear has been designed as an extension of the most successful and effective manipulative used to teach arithmetic—Base Ten Blocks. The Lab Gear is completely compatible with Base Ten Blocks—in fact, the two can be used in conjunction with each other to teach algebra, arithmetic, or both. The inclusion of blue blocks to represent variables allows the Lab Gear to be used at a higher level of abstraction. The sizes of the x-blocks and y-blocks were carefully chosen to prevent the confusion that arises when a block such as the Base Ten rod, having whole number dimensions, is used to represent a variable.

The Lab Gear is an extension of the development of algebra manipulatives that has taken place over many years. Zoltan Dienes, who was an early promoter of Base Ten Blocks, was the first to see their potential as a manipulative for algebra. Mary Laycock extended his ideas in work with multi-base blocks and popularized the "upstairs" representation of minus. Peter Rasmussen came up with the important idea of an x-block that is not a multiple of 1 and with the precursor of the corner piece.

The Lab Gear incorporates the best of its predecessors' designs, but it goes further, by including features that have advantages over other algebra manipulatives:

The corner piece: Helps organize the rectangle model of multiplication and division in two or three dimensions.

The workmat: Provides an environment used for equation solving and for operations with signed numbers.

A powerful combination of **two methods to represent minus** (the minus area on the workmat, and "upstairs"). This model is mathematically superior to the two-color model (see the section *Computing With Signed Numbers* on pages 579–591).

The use of **two variables** (x and y) adds flexibility and makes it possible to work with problems involving two variables, including the solution of simultaneous equations.

The use of **three-dimensional blocks** rather than tiles, which makes it possible to represent quantities such as x^2y and y^3 and to show the product of three factors.

All of these unique features work together to create a unified concrete environment in which to learn many concepts of algebra.

■ Grid Tools

This book often uses geometric topics as a source of data for algebraic generalizations. Much of this work is done with the help of three grid tools:

Graph paper: An important use of graph paper is as an environment in which to study perimeter and area, topics that lend themselves to interesting algebraic questions. Of course, graph paper also serves as a background for Cartesian graphing, and it can be used to construct function diagrams.

Geoboards: These tools provide another environment in which we study area. By including approximately one geoboard lesson in each of the first nine chapters, we gradually build the necessary background for a geometric introduction to proportion and slope, the concept of square root, and justification for the rules used in manipulating radicals.

Dot paper: While geoboards are initially very popular, students gradually realize that the same work can be pursued very well on dot paper, with less trouble and more accuracy. Moreover, dot paper can be cut to create *radical gear,* a manipulative for learning about the manipulation of radicals in Chapter 9.

■ Calculators

Calculators, computers, and other computational devices are everywhere. In all contemporary institutions, with the unfortunate exception of some schools, electronic methods are replacing paper-and-pencil computation.

■ Algebraic Symbols

The last tool we will discuss is the use of algebraic symbols. The manipulation of symbols used to be the nearly exclusive subject of the beginning algebra course. In this book, algebraic symbols still have an important place, but the emphasis is on the ideas they embody. Key structural rules, such as the distributive law and the laws of exponents, are important to understand, not only because of their importance in the manipulation of symbols, but also because they offer insights into the meaning of the operations.

We do not expect students to be able to understand algebraic symbols spontaneously. For most students, it takes a lot of work to get to the point of being able to use algebraic notation well. The work is essentially the process of abstracting structure from numerical data. This movement from the specific to the general is one of the essential activities of mathematics and science. Symbol sense is not assumed at the beginning of the course, but developing it is one of our main goals.

CONCEPTS

While symbol manipulation is a useful tool, accurate and/or speedy manipulation is no longer defensible as a central goal of the new algebra. Instead, the goal should be understanding of concepts. Tools and themes are the means, not the end. Their purpose is to help create a course where students can learn algebra concepts such as functions, numbers, variables, operations, equations, and more generally, mathematical structure.

As for mathematical terminology and notation, we have tried to navigate a middle course; too much jargon is intimidating and premature, but some ideas cannot be expressed precisely without using correct mathematical language.

The following is an overview of some of the major concepts in the book.

▶ Number Sense

To understand symbols, one must understand numbers. As a result, this book includes quite a lot of work with numbers, but the work is spread out through the year, and is geared to problem solving and real-world applications, not arithmetic algorithms. For example, we review percent in an interesting applied context, connecting it with the use of exponents, and we use graphs to throw light on the arithmetic of signed numbers.

The main extensions of students' number sense beyond the material they are likely to have seen before is in the work with scientific notation, and the work with square roots. We give these topics plenty of attention.

If some students have difficulties with basic arithmetic, help them use their calculators to solve problems. For example, you will be very popular if you teach students how to use the fractions-processing capabilities of their calculators. Many problems, interspersed throughout the book, directly address students' misconceptions about numbers. For example, their lack of clarity about multiplication and division by numbers between -1 and 1.

Avoid dwelling on arithmetic algorithms, such as signed number and fraction arithmetic, at the beginning of the school year. By the time students take algebra, most are happy to be done with their study of arithmetic, either because they already understand it, or because they have never understood it and see no reason to expect that they will. When discussing real-world problems, treat the necessary review of fractions, decimals, and percent with a light touch. Put the main emphasis on the algebraic structure of the problem and on discussion of the reasonableness of the numerical results. (If, however, you work on the rules of signed number arithmetic, you can use the reproducible Lab Gear-based lessons in the back of this book.)

▶ Function Sense

The concept of function is addressed throughout the book, not just in one lesson or chapter. We approach functions in many ways, through real-world applications, graphs, in-out tables, and function diagrams. Proficiency in using several representations of function and the ability to move from one representation to another are the foundation of a solid understanding.

We introduce composition of functions, inverse functions, and transformations of graphs, concepts that are usually postponed until Algebra 2 or pre-calculus. Because our approach is supported by effective tools, we have found these subjects to be well within the grasp of Algebra 1 students at an exploratory level.

Providing a strong foundation in functions and making these more advanced concepts understandable has a worthwhile pay-off: we use them to create a conceptual approach to many Algebra 1 topics that have been traditionally taught through the learning of rules. We use functions and their graphs to help students understand operations with signed numbers, equation solving, inequalities, the laws of exponents, square roots, and many other topics.

▶ Equation Solving and Inequalities

Traditional algebra courses often introduce the solving of linear equations too early in the semester. This forces students who have no sense of what variables are, much less equations, to memorize algorithms that have no meaning to them. Our approach is to spread the introduction of linear equations over the entire first semester, or even longer, in the case of a two-year course.

Instead of a standard procedure, we introduce a multiplicity of techniques.

Trial and error (also known by many teachers as guess and check): First and foremost, because it throws the most light on the meaning of the question, and empowers the students to seek the solution on their own.

The cover-up method: Mostly useful to highlight the concept of inverse operations.

Through graphing: This is particularly useful with the help of electronic calculators, and has the advantage of being useful for solving any equation whatsoever, even one like $x^3 + 2^x - 3/x = 456$.

With the Lab Gear®: This is strictly a transitional environment, intended to provide an opportunity for students to create their equation-solving techniques and rules.

By applying rules of algebra: This is the closest to the traditional technique, though we never impose a particular sequence, leaving it up to students to develop their own strategies.

As for inequalities, we do some work with them, but limit ourselves to trial and error, graphing, and Lab Gear approaches. We believe that it is counter-productive to teach rules for solving inequalities in Algebra 1, since students almost always remember them incorrectly. Instead, we provide several possible approaches and, in optional exercises, hint at some rules that students might develop for themselves.

▶ Proportional Thinking

The absence of experience using proportional thinking in traditional algebra courses means that students studying similar triangles in geometry, and later applying what they learned in trigonometry, are under-prepared. Similarly, students in science classes, where proportional thinking is essential, need solid grounding in the mathematics of proportions. We address this need by using geometric and real-world contexts in lessons that are spread throughout the book. Direct variation is emphasized early in the book, and approached from several different angles. We also cover unit conversion from a mathematical standpoint, a topic that is usually relegated to science classes. We contrast comparing by ratio to comparing by difference, an idea which is important in many applications, but usually overlooked in algebra courses.

▶ Mathematical Structure

By this we refer primarily to an understanding of the algebraic structures underlying the real number system, meaning especially an understanding of the operations and their relationships to each other. This includes the distributive law, the laws of exponents, and the rules for operations with radicals. We avoid the overly abstract approach of the *new math,* and instead, concentrate on developing students' understanding through examples and models.

We also put this understanding in a broader context through some optional lessons on abstract algebra (via specific examples well within students' reach) and the relationship between different types of numbers (real, rational, irrational, integers, natural) and different types of equations.

▶ Symbol Sense

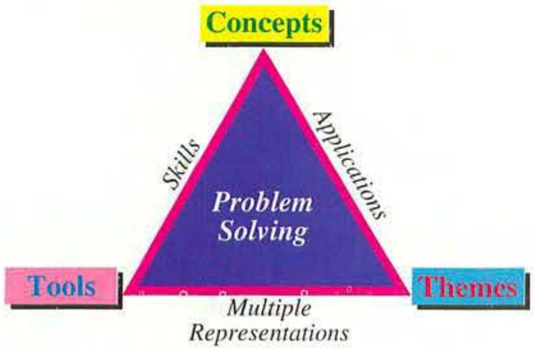
We emphasize some of the topics about symbols that were not taught effectively in the traditional course such as the gradual process of replacing numbers with variables when thinking of a real-world problem, and the meaning of parameters, especially in certain basic forms such as $y = kx$, $xy = k$, $x + y = k$, $y = mx + b$.

As students' understanding of algebraic concepts deepens, they are gaining symbol sense—an appreciation for the power of symbolic thinking, an understanding of when and how to apply it, and a feel for mathematical structure. Symbol sense is a level of mathematical literacy beyond number sense, which it subsumes. It is the true prerequisite for further work in math and science, and the real purpose of the new algebra course. As the NCTM puts it, Algebra is the language through which most of mathematics is communicated. It also provides a means of operating with concepts at an abstract level, and then applying them, a process that often fosters generalizations and insights beyond the original context.

ACHIEVING MATHEMATICAL POWER

Tools and themes create an environment in which students are empowered and motivated, when problem solving, discovery, and cooperative learning can thrive, and where skills can develop naturally and in context. Our goal is to help students develop the mathematical skills necessary for future math courses as well as for the world of work.

▶ How Tools Help



The multi-tool approach has four advantages over more traditional methods.

Access: By providing immediate feedback, these tools make it possible for all students to get involved with significant mathematical concepts. This is not to say that tools make algebra easy. Algebra is abstract and difficult for most students. It takes persistent effort to develop mastery. But tools do make algebra accessible to all who are willing to work at it.

Discourse: The tools also facilitate the transition from a traditional class format into one where discovery learning, problem solving, and cooperative work are the norm. Instead of the teacher's authority being the sole arbiter of correctness, tools make it possible for students to use reasoning and discussion about a concrete reference as a way to judge the validity of mathematical statements.

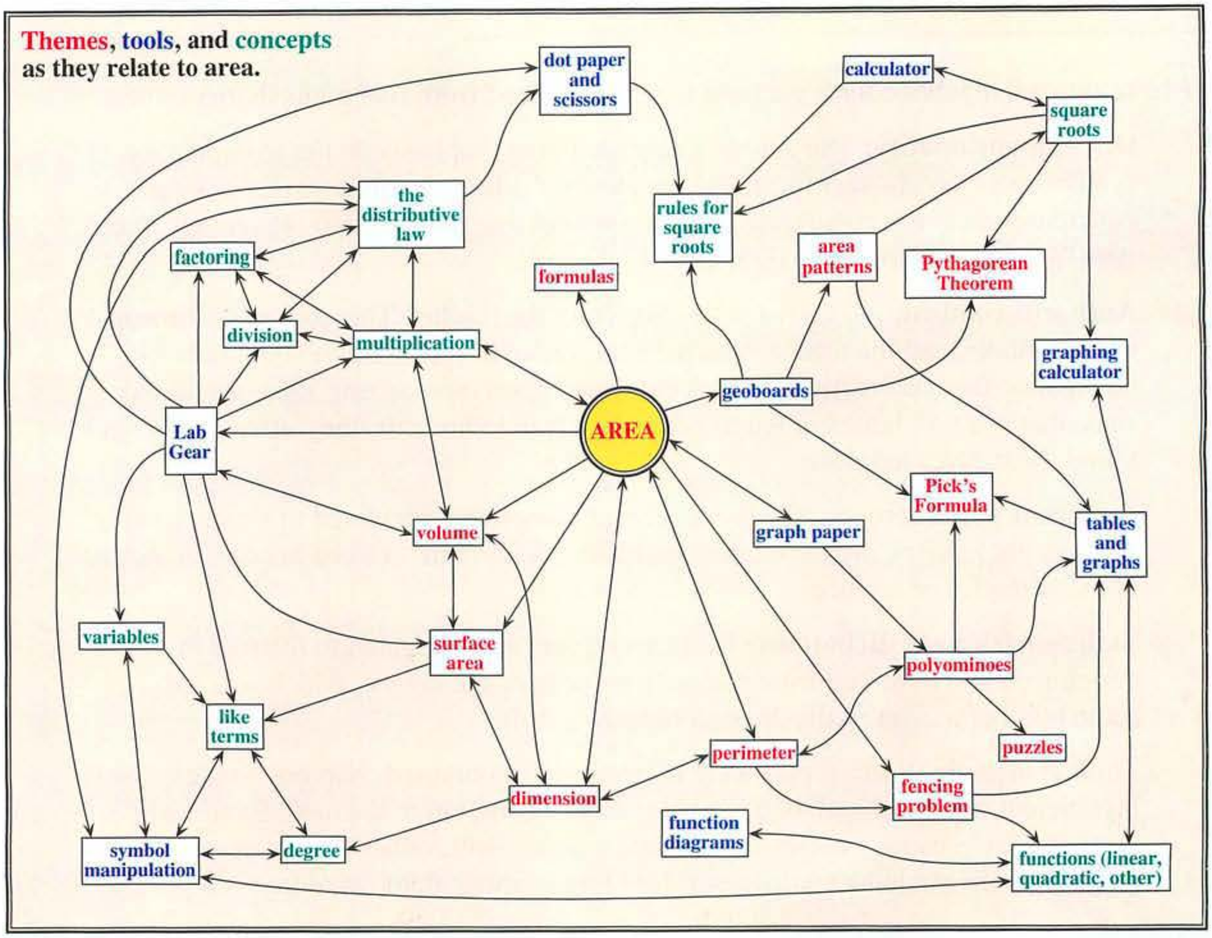
Independence: As students work with tools over time and develop more and more understanding of the concepts of algebra, they have less and less need of certain tools (such as manipulatives or function diagrams), which have merely served as a bridge to understanding abstract ideas. On the other hand, they become more sophisticated users of other tools (such as calculators and electronic graphing devices), which will remain useful throughout their mathematical careers. In both cases, the students are more self-reliant, and therefore, more self-confident.

Multiple Representations: There is a synergy in the interaction of math tools. For example, a student who has thought about square roots in a multidimensional way, with the help of geoboards, dot paper, radical gear, calculators, and graphing calculators, has much more depth of understanding, particularly if the relationships among the representations have been made explicit, than a student who has practiced only disembodied operations with radicals.

In a multi-tool environment, students will develop specialties. Some will be Lab Gear experts, while others will be most comfortable with the use of calculators or function diagrams. This is normal, and cooperative learning allows all students to benefit from each other's strengths.

Finally, this book does not depend on any one tool. If at first you are uncomfortable with function diagrams, or with the Lab Gear, you should de-emphasize those lessons until you have become better acquainted with the tool. Of course, you will get the most out of this book if you take some time to familiarize yourself with all the tools.

▶ Algebra as a Web



Themes, tools, and **concepts** are interwoven in a complex web. This map was constructed by starting from area, a sub-theme of measurement, then showing the connections between algebra topics, between algebra and geometry, and between algebra and the real world. The map does not include the whole book, but it does reveal how our approach goes about making these connections. In the traditional curriculum, which assumes an arbitrary sequence within algebra, these connections are masked, and the geometric connections aren't made at all.

Contrast it with the corresponding map for the traditional course shown here.

Area in the Traditional Algebra Course

CONCLUSION

*T*he traditional algebra course we used to teach suffered from five main shortcomings.

One-dimensionality: The course's overwhelming emphasis on the manipulation of symbols was too abstract for many students; for others, it was boring. Lacking a concrete context for communication, classes became divided into two groups: those who "got it" and those who didn't.

Authoritarianism: All knowledge came from the teacher. The goal was to manipulate symbols, and the teacher was the sole source of information about how to manipulate them correctly. Students depended upon memorizing algorithms, and since that is a task better suited to computers than to humans, they often forgot, and found themselves helpless.

Apparent pointlessness: The work seemed completely unrelated to situations students might realistically encounter outside the classroom, or even in other branches of mathematics or science.

Skills/enrichment dichotomy: Problem solving was relegated to the role of enrichment and divorced from the main purpose of the course, which was the acquisition of narrow skills through repetitive drill.

Topical organization: Topics were taught in self-contained chapters. Students had insufficient time to absorb one new idea before going on to the next. Even word problems were usually constructed to test a single skill, rather than to draw on and to exercise the students' entire reservoir of mathematical knowledge.

These shortcomings were both curricular and pedagogical. They could not be addressed by piecemeal change—we needed a revolution.

▶ Strategies for Change

As you embark on the process of making changes, you may run into resistance from many quarters. Students may feel that the course involves too much work, and particularly, too much writing. Some even complain it requires too much thinking. Parents may worry because the course is different from the one they took. More significantly, students, parents, and colleagues may be concerned about whether students will be prepared for the next course, which may be quite traditional. Some may feel anxiety about the suitability of the course for strong (or weak) students.

We have tried to respond to these concerns in this introduction. In these closing comments, we will make a few suggestions about strategy.

Start with the changes in pedagogy and be more gradual about making changes in content. As students become more involved in their own learning, and the results are obvious to all, and as national, state, and local requirements change, it will become easier to implement content changes.

Make sure your stronger students are challenged. They, and their parents, are among your most important allies in the process of change.

If you have trouble with certain lessons or types of lessons, de-emphasize them, and fall back on more familiar material.

Work closely with other teachers who agree with you about the need for change. In particular, try to find other teachers who use this book, and help each other out.

As your students become more skillful at writing, have them polish reports and projects and display them on bulletin boards. Publish outstanding write-ups on real-world problems in school or PTA newsletters.

▶ The Rewards

Making change is difficult, but rewarding. Here are some statements from teachers who used preliminary versions of this book.

This book is so rich, there is so much to do, you cannot do it from A to Z. You start investigating something that seems trivial, that all students can get into, and soon you are challenging your top students.

It's so concrete, it's a whole new way of doing algebra. When someone makes a statement, you can say, "Show me."

The only problem I have with class control is that everyone wants to participate.

Students get so involved in a problem that they won't want to leave at the end of the period. That used to happen in art class, not math class.

The book supports discovery. It takes the teacher away from being the center of attention. I feel less pressure; I'm just another participant in the learning environment. I can step back and be more aware of what's going on and where the kids are, so I can be more helpful.

This book meets students where they are and brings them along from there. They develop so much confidence. We ask them to do things they've never done before, but since they're used to that, they just plunge in confidently.

A lot of problems don't have just one answer. I like that. It means that a lot of students can get into it and feel as if they're doing something right. The book supports conjecturing. Kids are not so attached to having the one "right" answer.

Using the Lab Gear helps us appeal to all kinds of students. Several students told me, "If it weren't for this, I'd never understand algebra." Some of them even bought their own set of Lab Gear to use at home. A few resisted the Lab Gear, but then we'd find some of those same students going to get it during a test! It really helped when we set up the classroom as a lab, so that the tools were always accessible for those who needed them.

The hardest thing about using this book was getting used to writing and explaining. So often at first when the book said, "Explain," students would ask, "What does this mean? What do they want?" I had to really think about how to answer them and I realized that it helped me understand, too! I like all the writing. Writing is hard for me, but when I write about something, it helps me internalize the ideas and really understand them. It does the same for the students.

It took us a while to learn how to incorporate writing, but it was well worth the time and effort. Our students had to get used to writing and to our standards for good writing, but once they got the idea they were so proud of their work. We learned to be careful not to assign too much, and to give them enough time to work on it so that they could do high quality work. They put a lot of effort into their Thinking/Writing assignments. After a while, those assignments generated a lot of enthusiasm.

Dear Parent,

This book is different from the book you used if you took algebra. It certainly is different from the books we used. We have taught from many algebra textbooks over the years, and are well acquainted with the traditional algebra course. The course had many problems: there were many Ds and Fs, and even students who got good grades often did not really understand what they were doing. In addition, the development of calculator and computer technology has made it imperative to change the emphasis of the course. Moreover, as a profession, math teachers now have a better understanding of how students learn.

This book is based on three big ideas, which have been guiding principles in our teaching:

- In order to learn to reason flexibly and independently about the abstract concepts of algebra, students need tools to think with. These tools should be designed to support students' work with the main ideas of algebra: variables, operations, equations, functions, and so on. We use manipulative, electronic, and old-fashioned pencil-and-paper tools.

- Learning mathematics should be based on solving interesting problems. Students' skills develop best if they are given an interesting context to practice them in. Look through the book at the wide variety of problems we address: air travel, get-rich-quick schemes, telephone billing plans, children's growth rates, making cranberry-apple juice, car and bicycle trips, and on and on.

- Most students will not remember concepts if they are explained once or twice by a teacher and practiced in isolation over a short period of time. Students must be involved in their own learning, and have experience with ideas in many forms and formats over an extended period of time. They must experiment, conjecture, discover, and write about what they are thinking. In this book, important ideas are returned to over and over, and much work is expected of the student — hard work, but work that is more varied and interesting than the traditional drill and practice.

After using this book, your child will be exceptionally well prepared for future courses, because we have made a point of giving extra emphasis to the areas that are most important to the rest of secondary school math and science: square roots, proportions, scientific notation, functions, and symbol sense. In addition, the emphasis on thinking, communication, and writing skill will help across the whole curriculum.

If you have any questions about this course, we are sure your student's teacher will be glad to help answer them. The biggest help you can provide is to make sure that your student does algebra homework every day.

Sincerely,

Anita Wah and Henri Picciotto