Construction: Teachers' Notes

Introduction

This unit is slightly edited from the one that is taught in the second term of Math 2 at the Urban School of San Francisco.

Like proof, construction is about logic: given a starting configuration of points and lines, how does one correctly create a target figure if one is only allowed a given set of tools? There is a puzzle-like quality to these questions, and tough construction challenges are difficult even for the best students. Like many appropriately chosen puzzles in the classroom, they can be very motivating to a wide range of learners.

There is an ancient tradition of compass and straightedge construction, of course, and this approach definitely fits in that history. However I have found that I get more educational mileage by diversifying the toolbox. I still use the compass to make circles, to copy line segments, and to bisect segments and angles, but I prefer patty paper (tracing paper) to copy angles. Beyond a certain point I require the use *interactive geometry*: computer-aided construction tools to pursue a range of challenges. For many years, I used Cabri, and appreciated its elegance and mathematical depth. Now, I use GeoGebra, which works largely the same way, with different strengths and weaknesses.There are of course other interactive geometry software applications, and these lessons can be done with any of them — Geometer's Sketchpad, Cinderella, etc.

There are a number of important results in the unit: the perpendicular bisector of a segment, the angle bisector, the various centers of triangles and their properties, tangents to circles, inscribed and circumscribed circles, and so on.

This work can lead to a geometric introduction to the conic sections.

If Soccer Angles (*Geometry Labs,* Lab 1.10) was done previously, it can be followed up towards the end of this unit with a construction of the point with the optimal shooting angle.

Caution: it is imperative you work through the sheets yourself before introducing them to your class.

Bisector Theorems

It is very effective to precede this lesson with a quick outside activity. A gym is a good choice, but any large space will do.

Ask students to arrange themselves so that they are:

0. Equidistant from a point (a given student)
1. Equidistant from the endpoints of a segment (two students)
2. Equidistant from the rays of an angle (for example two intersecting lines on the basketball court, or two consecutive edges of a lawn, etc.)

Each time, have the students discuss where they had to place themselves. (For more information on this activity, see <www.MathEducationPage.org/kinesthetics>.

Basic Constructions

For the first few problems, just tell the students how to do it. (It can be found in just about any geometry textbook.) For the remaining problems, give generous hints.

Hints:

7, 8. Start with the diagonals. (See *Geometry Labs*, Lab 6.3: Making Quadrilaterals from the Inside Out)

9. This problem is mostly here to keep students who worked fast up to this point busy while their classmates catch up.. Here is a solution students are not likely to find: draw any perpendicular L to the original line D. Perpendicularly bisect the segment connecting the intersection of L and D with the original point F. The intersection of the perpendicular bisector and L has the required property. Doing this many times, one gets points on a parabola.

Circles Through Points

This page and the following ones assume students have already been introduced to interactive geometry software.

Encourage students to help each other.

Tangent Circles

Be stingy with the hints until #6-8. For #6, see the hint for #9 of **Basic Constructions**. For #8, follow the same logic, starting with a diameter of the circle for line L. For other, simpler constructions, see <www.MathEducationPage.org/conics>.

Tangent Lines

Prerequisite: the theorems about tangents to a circle.

#7-8 are very difficult.

Lines in a Triangle 1

This is where we introduce the concurrence theorems.

Prerequisites: the theorems about bisectors, and their converses.

You will need to prove #1, as it is almost certain that students will not be able to do this on their own.

Lines in a Triangle 2

Prerequisite: the midsegment theorem.

After #2, you can ask students to find the centroid of a cardboard triangle, and have them balance the triangle on a finger. It is not necessary to do a formal construction: just use a ruler to find the midpoints.