

Focus on Function Diagrams — Teacher Notes

This activity is centered on the figure "Sixteen Function Diagrams." (See page 3 of this document.) For a better-quality version of the figure, use the separate PDF file titled **sixteen-fds.pdf**

Core Sequence: 1-14, 16-22

Suitable for homework: 1-3, 12-22

Useful for Assessment: 3, 10, 16, 20, 22

What this Lesson is About:

- Review of function diagrams for the form $y = x + b$
- Rate of change
- Preview of slope-intercept form

This is a substantial lesson, which involves a lot of work and effort. It is worth it, because it lays the groundwork for a full understanding of slope and intercept of linear functions.

Review: Parallel-Line Diagrams

This reviews function diagrams for functions of the form $y = x + b$. It is easy, and you can use the opportunity to review the function diagram terminology that will be useful throughout this lesson: in-out lines, x and y number lines.

The Focus

The exploration is important not only to understanding this lesson, but to understanding linear functions. Make sure there is plenty of class time to work on it, preferably in groups. Follow up group work with whole-class discussion.

If students have no idea how to begin, you may suggest they make in-out tables based on the diagrams, and try to figure out the patterns from that. You may also encourage them to start with the diagrams in the middle row, which are substantially easier than the other ones, and can provide a good starting place.

An in-depth discussion of the Exploration will make the rest of the lesson much easier, and perhaps unnecessary.

Magnification

One of the most useful features of the function diagrams of linear functions is the fact that they make the rate of change very visually clear. This section focuses on this visual evidence of "magnification" of the change in x that leads to the change in y.

The m Parameter

The main point of this section is that the magnification can be seen explicitly in the equation, if it is written in $y = mx + b$ form. (Could this be the reason why the letter m is used, and not a?)

While it is not important for students to memorize the information about the relationship between the position of the focus and the magnification, that discussion should help reinforce the visual sense of what magnification is about.

Students should be able to do #10-13 by discussing it in their groups and looking at the sixteen diagrams in the figure. If they find this difficult, you may lead a class discussion, perhaps based on some function diagrams on the overhead projector.

The answer to #14 is based on the fact that if the focus is far to the left, m is a little greater than one. If the focus is far to the right, m is a little less than one. If the focus is “at infinity”, m is exactly equal to one. So the idea of the focus at infinity is consistent with the finite case.

Rate of Change

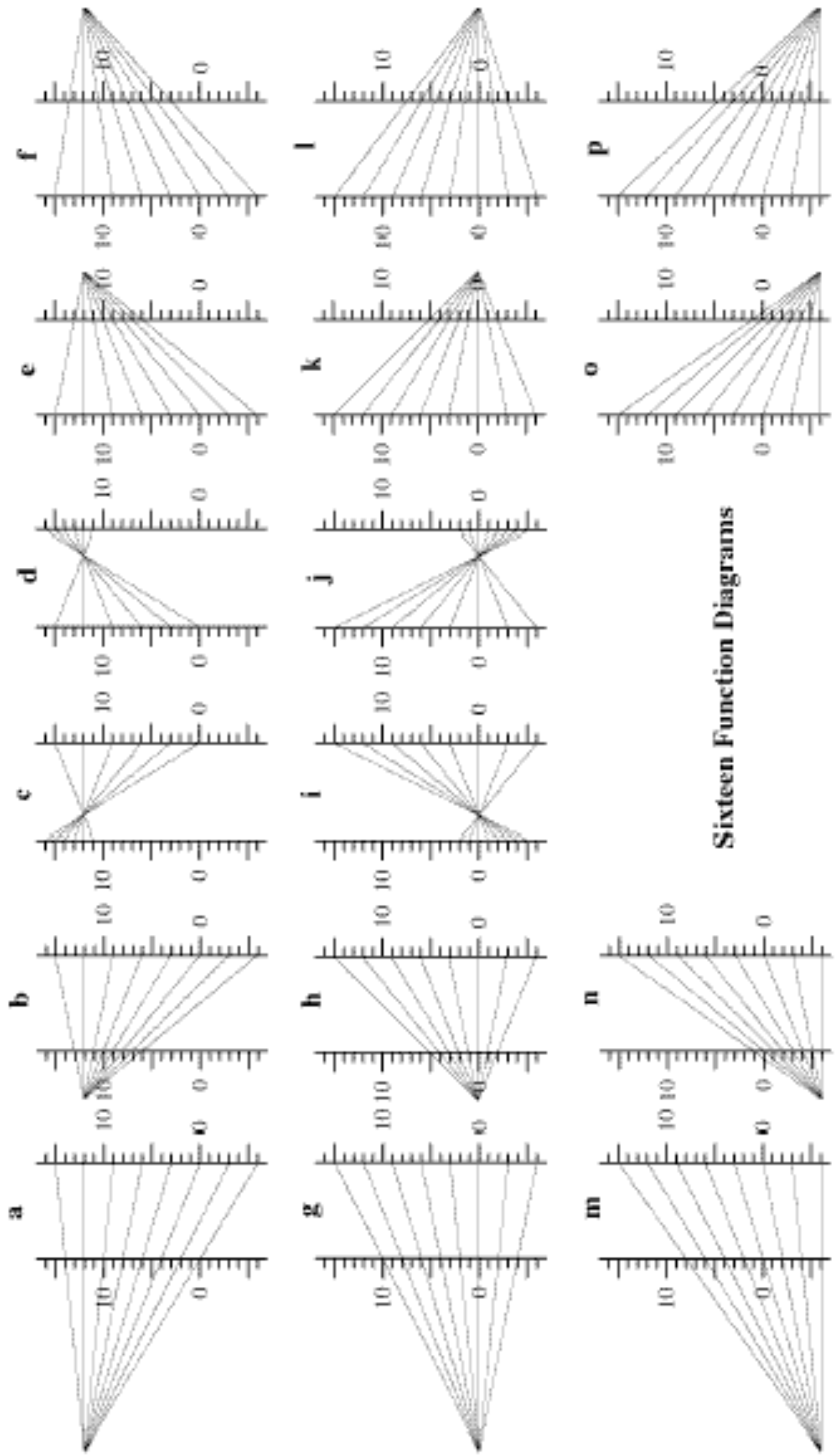
This short section serves only to introduce and justify the words *rate of change* as another name for magnification. (The word *slope* also refers to the same concept, but in the context of Cartesian graphs. It should of course also be introduced, though definitely not on the same day as magnification.)

The b Parameter

The relationship between the b parameter and the position of the focus is far from obvious when b is not 0. The only way to really see it is to remember the meaning of the in-out lines, and what happens when multiplying by zero.

y = mx + b

This wraps up the lesson. If some of your students have not completely mastered the ideas, you can still move on. You will be able to refer back to this lesson while teaching about the Cartesian graphs of linear functions. Out of the comparison of the two approaches (function diagrams and Cartesian graphs), your students should be able to develop a solid grasp of linear functions.



Sixteen Function Diagrams

Focus on Function Diagrams

You will need: graph paper

Review: Parallel-Line Diagrams

1. a. Draw a function diagram such that its in-out lines are *parallel* and going uphill (from left to right).
   b. Find the function corresponding to the diagram, using an in-out table if you need it.

2. Repeat the previous problem with parallel in-out lines going
   a. downhill b. horizontally

3. € For the functions you created in this section, when x increases by 1, by how much does y increase? Does it depend on the steepness of the lines? (To answer this, compare your functions with other students'.) Explain.

The Focus

Definition: If an in-out line is horizontal, its input is called a *fixed point*. For example, both x and y = 12 in diagram (a), so 12 is a fixed point for that function.

4. What are the fixed points for functions (b)-(h)?

Definition: In-out lines can be extended to the left or right. If all of them meet in a single point, that point is called the *focus*.

The attached sheet shows sixteen function diagrams that have a focus.

5. **Exploration.** Consider the function diagrams on this figure. For each one, find the function. You may split the work with other students. Describe any patterns you notice. If you cannot find all the functions or patterns, you will get another chance at the end of the lesson.

Magnification

6. Look at diagram (h) in the figure. By how much does y change when x increases by:
   a. 1 b. 2
   c. some amount A

In function diagrams that have a focus, changes in y can be found by multiplying the changes in x by a certain number, called the *magnification:*

(change in x)·(magnification) = (change in y)

7. a. What is the magnification for (h)?
   b. What other diagrams have the same magnification?

Rule: If y decreases when x increases, the magnification is negative.

8. For which diagrams is the magnification equal to -3? (If x increases by 1, y *decreases* by 3.)

9. Find the magnification for each diagram in the figure. Note that the magnification can be positive or negative, a whole number or a fraction.

The m Parameter

You probably noticed that all the function diagrams above represent functions of the form y = mx + b. It turns out that this is always true of function diagrams with a focus. As you may remember, the letters m and b in the equation are called *parameters*.

10. € Look at the equations you found in the Exploration. What is the relationship between the magnification and the m parameter in those equations? Explain.

11. If you move the focus of a function diagram up, how does it affect the value of m? How about if you move it down?

12. Where would the focus be if m was:
    a. a negative number
    b. a number between 0 and 1
    c. a number greater than 1

13. What is a possible value of m be if the focus is:
    a. half-way between the x and y number lines
    b. between the x and y number lines, but closer to x
    c. between the x and y number lines, but closer to y

14. What is a possible value of m be if the focus is:
    a. far to the left of the x number line
    b. close to the left of the x number line
    c. close to the right of the y number line
    d. far to the right of the y number line

15.* In some parts of mathematics, parallel lines are said to meet at a point that is "at infinity". In that sense, parallel-line diagrams could be said to have a focus at infinity. Is this consistent with your answer to the last problem? Explain.

Rate of Change

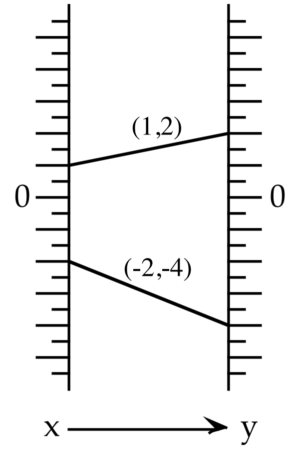
Once again, look at the diagrams (a)-(o).

16. € On each diagram, *as x increases,* follow y with your finger. For what values of m does y
    a. go up? b. go down?
    c. move fast? d. move slow?

The magnification is often called the *rate of change*.

17. What is the rate of change if y increases by 3 when x increases by
    a. 1 b. 6 c. -10

The b Parameter



Two in-out lines are shown in the above diagram. Each one is labeled with a number pair. The first number in the pair is the input, and the second number is the output.

Notation: Any in-out line can be identified by a number pair. From now on, we will refer to lines on function diagrams this way. For example, the line connecting 0 on the x number line to 0 on the y number line will be called the (0,0) line.

18. What can you say about the b parameter if the focus is on the (0,0) line?

19. Look at diagram (n). Its equation is: $y = 3x + 12$.
    a. Name the in-out lines that are shown.
    b. Check that the pairs you listed actually satisfy the equation by substituting the input values for x.
    c. Among the pairs you checked was (0,12). Explain why using 0 as input gave the b parameter as output.

20. € In most of the diagrams (a)-(o), there is an in-out line of the form (0,___). How is the number in the blank related to the b parameter? Explain.

y = mx + b

21. If you did not find all the equations for the function diagrams (a)-(o), when working on Problem 4, do it now. Hint: you may use what you learned about magnification and about the (0,___) in-out line.

22. **Summary.** Write about what you learned about function diagrams, the fixed point, the focus, magnification, and the parameters m and b. Also mention parallel-line diagrams.