Geometry Labs

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Finally, I'd like to thank the authors of the Elementary Science Study at the Educational Development Center, whose math units *Tangrams* and *Pattern Blocks* awakened in me an interest in doing math with manipulatives when I was a beginning teacher, many years ago.

Henri Picciotto

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Introduction

About This Book

This book is a collection of activities in secondary-school geometry. Most of the activities are hands-on and involve concrete materials. Many of them have enough depth to provide excellent opportunities for discussion and reflection about subtle and important ideas. Like others of my books, this one is not geared to a narrow track such as "honors," "college-bound," "regular," or "remedial." Most labs were developed for heterogeneous groups that include some strong students, but you should be able to find plenty in here to use in just about any classroom.

This is not a textbook, and I do not claim that it offers a comprehensive treatment of secondary-school geometry. However, it addresses many essential ideas and can be a substantial part of math classes at many levels:

- *Middle school teachers* will find many labs that help prepare students for high school geometry by getting them to think visually and to become familiar with fundamental concepts, figures, and vocabulary. It is unfortunate that much of the curriculum currently available for middle school geometry consists of rote activities built around a few results when there is a wealth of possibilities.
- *Teachers of geometry* courses, whether traditional, inductive, or technology-based, will find many labs that approach key topics in their curriculum from a different point of view. In some cases, you will find, as I have, that the lessons in this book can replace the corresponding ones in your textbook. Other labs can be used to *preview* or *review* material that you teach in more traditional ways.
- *Teachers of integrated mathematics* courses at any secondary-school level will have no trouble finding suitable labs in this book, either to support the geometry component of their courses, or to enrich it.
- Finally, *trigonometry teachers* will find interesting material in Section 11. The approach I pursue there should make trigonometry material accessible to younger students.

Here is a rough estimate of grade levels appropriate for sections of this book:

Sections 1–8: grades 7–11

Sections 9–11: grades 9 and up

In some cases, I included lessons whose purpose is to introduce an idea that is a prerequisite to a subsequent lab. While these can usually be skipped in a class that is also using a geometry textbook, they are included for the use of middle school teachers or for teachers of integrated mathematics courses whose textbooks sometimes do not include those concepts. Such labs are designated as "getting ready" activities in the Teacher Notes.

Geometry and Proof

Formal proof has a central role in high school mathematics. Traditionally, proof has been introduced in the geometry course, but, unfortunately, this has not worked as well as many of us would like. In many traditional courses, the first proofs are of self-evident results like "the angle bisector divides the angle into equal angles," which is a sure way to baffle beginners. In other courses, proof is completely avoided.

A successful introduction to proof has to be rooted in rich mathematical content and in the use of discourse and reasoning. This book provides an enormous supply of rich mathematical content and opportunities for discourse and reasoning. While it does not take the next step of helping students learn to write formal proofs, it does set the stage for such learning. At the Urban School, we introduce formal proofs in the second semester of the course, building on the foundation laid by many of the labs in this book.

Still, important as it is, I don't see the introduction of proof as the only reason for teaching geometry:

- There is plenty of geometry content that is of great importance to further work in mathematics. I am thinking of topics such as measurement, distance and the Pythagorean Theorem, and similarity and scaling, all covered in the last four sections of this book.
- Geometry is surely an area in which the aesthetic appeal of mathematics is most clearly evident. I certainly hope that you will find time for the sections on symmetry and tiling, which will appeal to the artist in each of your students.
- Finally, geometric puzzles have a fascination for many. I have mined recreational mathematics for material that is both entertaining and educational, and I have inserted puzzles and other visual challenges (many of them original) throughout this book.

Using Manipulatives

Almost all the activities in this book are based on the use of manipulatives. This is not because I believe that there is some sort of magic that guarantees learning as soon as students are manipulating something. I have been involved with manipulatives in math education for over 25 years and have no such illusions. Still, I do believe that manipulatives can help in the following ways:

- Manipulatives can motivate students, particularly if they are used in a thought-provoking, puzzle-like way. In this book, I have tried to choose activities that are intrinsically interesting and that appeal to a range of students.
- Manipulatives can be the spark for significant discussions, both at the small-group level and with the whole class. In addition to the exercises on the worksheets, most labs also include Discussion questions that can lead to stronger understanding for all.

To get the greatest mathematical payoff from the use of the manipulatives, it is essential that you make explicit connections between what is learned in this context and the broader goals you have in your course. I have included suggestions for this in the Teacher Notes that accompany each lab.

Using the Discussion Questions

The Discussion questions tend to be more difficult, more general, or deeper in some ways than the rest of the lab. Generally, they are intended to follow the lab and can be used to spark whole-class or small-group conversation. However, be flexible and alert. Often, the Discussion questions can be used smack in the middle of the lab, when students are ready to think and talk about them. In many cases, the Discussion questions make excellent prompts for in-class or homework writing assignments. Written responses can serve as the basis for a richer discussion or as a valid assessment tool.

During most labs you should help students mostly with hints and nudges and encourage them to seek help from each other. However, it is likely that you will need to play a more directive and assertive role in the discussions. Do not be afraid to explain difficult ideas to students—once they know what the questions are. One way to make a lecture ineffective is to try to answer questions students don't have. The labs and Discussion questions that follow will generally raise the appropriate questions and prepare students to hear your explanations.

Using the Teacher Notes

The Teacher Notes are designed to help you prepare for the labs. They start with a list of prerequisite concepts, if there are any. This list can be crucial because you may be choosing individual lessons here and there rather than using the labs sequentially from beginning to end. The prerequisites are either general geometric ideas that are probably already a part of your course, or they are a specific lab or labs from an earlier part of this book.

The Teacher Notes also provide help with timing issues. In the never-ending conflict between "covering" and "discovering," you need to keep a balance and avoid going too far in either direction. Since, for most of us, the former

has a tendency to dominate our thinking, I would encourage you to not rush through the labs. Most labs should fit in one class period, but for those that don't, it is better to allow two periods or use longer periods if you have those somewhere in your schedule, even if the consequence is that you have to limit yourself to doing fewer labs. I have tried to indicate the labs that are likely to take a long time, so you can plan accordingly. Alas, it is often the more valuable labs that take the most time.

One more thought about time: Be aware that getting to closure on all the Discussion questions could lengthen some labs more than you can afford in a single period, so you will have to choose which of those questions you want to address in a thorough manner. On rare occasions, there are questions in the body of the labs that are potentially very difficult or time-consuming. I tried to flag those in the Notes and Answers.

Tools and Concepts Overview

This table shows which sections use which tools, to teach which concepts. Use it in deciding what to teach, what manipulatives you will need, and so on.

"Shape" includes the definition, recognition, and understanding of various geometric shapes, plus concepts such as convexity and congruence. "Measuring" includes perimeter, area, surface area, and volume, as well as distance and square root.

	Pattern Blocks	**Template**	**CircleTrig Geoboard**	**Tangrams**	**Cubes**	**Mirrors**	**11 × 11 Geoboard**
Angles	1, 3, 5, 7	1, 3, 5, 6, 7	1, 3, 5				11
Shape	3	6, 7	3	2	4		6
Symmetry	2, 5	2, 5, 6	5		4	5	
Dimension		6			4, 10		
Tiling	7	7			7		
Measuring	9	9	9	10	8		8, 9
Similarity			11	10	10		10, 11
Trigonometry		11	11				

Some tools are always expected to be available. These include a calculator, a compass, a ruler, a protractor, and assorted types of papers: unlined paper, graph paper, and grid papers and record sheets you can duplicate from the back of this book.

If you have access to computers, I strongly recommend that you make use of them in teaching geometry. Programming languages with turtle graphics, such as Logo and Boxer, and interactive geometric construction programs, such as The Geometer's Sketchpad® and Cabri II™, offer many opportunities for great labs. This does not conflict with the "lower" technology of manipulatives: Each type of lab has its uses and its place in a full geometry program.

Specific Notes on Some of the Manipulatives

I recommend you buy the complete kit designed for this book. Two items in the kit—the CircleTrig Geoboard and the drawing template—are unique. If you already have a collection of commonly used manipulatives, you may get by with what you have, perhaps supplemented by a separate purchase of the unique items. Study this list to see what you'll need:

Pattern Blocks: Pattern blocks are very popular with students and pretty much indispensable to key labs in this book. They were invented in the 1960s by the Elementary Science Studies and can be found in most elementary schools. My students sometimes affectionately call them "kindergarten blocks," and no, they do not find them offensive.

Template: The *Geometry Labs* drawing template will allow your students to conveniently draw pattern blocks, regular polygons, and a decent selection of triangles and quadrilaterals. In addition, the fact that it incorporates a straightedge, an inch ruler, a centimeter ruler, a protractor, and a circle shape makes it a useful tool to have at all times in math class. If you already have other templates that fulfill some or all of those functions, you may be able to use those instead. The first time you use the template, ask students to trace all the figures on a piece of unlined paper to serve as a Template Reference Sheet. Every time you use a new figure in a lab, have students label the figure on the sheet.

CircleTrig Geoboard: The CircleTrig Geoboard I designed for *Geometry Labs* has pegs every 15 degrees around the circle and every 2 cm around the perimeter of the circumscribing square. The engraved degree marks on the circle and millimeter marks on two sides of the square make it the most versatile circle geoboard around, extending its use to trigonometry topics. The 11×11 square geoboard on the other side of the board makes this the best general-purpose geoboard you'll find. You can use a classic circle geoboard, which is much smaller but adequate for Sections 1–10. For Section 11, you're better off with the CircleTrig board, but you can survive by duplicating page 245.

Tangrams: Tangrams are a classic geometric puzzle, with a long history in and out of the classroom. They are used only in two sections, Sections 2 and 10.

Cubes: Cubes are useful as multi-purpose tools in a math classroom. If you have cubes other than the ones supplied in the *Geometry Labs* kit, you probably can use them for most labs that require cubes. Interlocking cubes are definitely preferable to non-interlocking cubes, and cubes that attach on all six faces are preferable to those that don't.

Mirrors: Mirrors are used only in Section 5. If you have other mirrors or some equivalent, those will work just as well.

11 × 11 Geoboard: Crucial in the latter half of the book, this geoboard makes an important connection with algebra and the Cartesian plane. The 5 × 5 geoboard is not really adequate for the purposes of these labs, but dot paper is a passable substitute.

Related Publications

More labs along the lines of the ones in this book can be found in the *Pentominoes* and *Super Tangrams*™ books, which I created for Creative Publications. They are suitable for both the middle school and high school level. See also: <www.MathEducationPage.org/puzzles>.