Infinity: An Alternate Elective After Algebra 2

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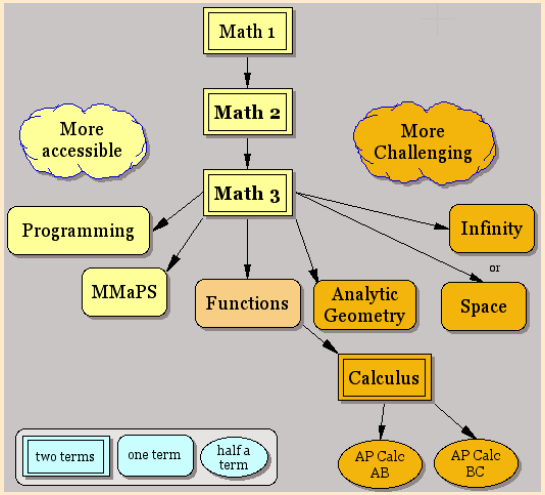
Math Courses



not tracked / grade levels & acceleration

Infinity overview

Who takes the class

Four topics

Readings

Algebra review

Computer tools

Juniors, before Calculus

Seniors, instead of or in addition to Calculus

not all superstars

Infinity overview

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Infinite sets

Proof

Chaos

Fractals

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Galileo,
Jorge Luis Borges,
Douglas Hofstadter,
Martin Gardner,
Lewis Carroll,
James Gleick,
Scientific American,
...

Infinity overview

Who takes the class

Four topics

Readings

Algebra review

Computer tools

prime numbers,
algebraic fractions,
similarity,
proportions,
sequences and series,
iteration,
logarithms,
complex numbers,
...

Infinity overview

Who takes the class

Four topics

Readings

Algebra review

Computer tools

Fathom

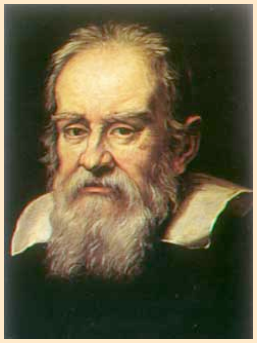
Boxer

Boxer is a key part of the course, but I won't show that part.

Infinite sets
Proof
Chaos
Fractals

Galileo

1564-1642



Get two people to read the dialogue

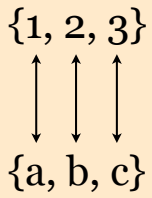
Infinite sets
Proof
Chaos
Fractals

Georg Cantor

1845-1918

Equivalence

Two sets are equivalent if their elements can be put in a one-to-one correspondence. Example:



Equivalence

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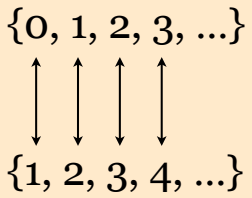
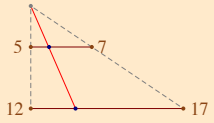


Equivalence

Two sets are equivalent if their elements can be put in a one-to-one correspondence. Example:

[5, 7] and [12, 19]



Cabri file: intervals

Equivalence

Two sets are equivalent if their elements can be put in a one-to-one correspondence. Example:

$[0,\infty)$ and $(0,\infty)$

$$f(x) = \begin{cases} x+1 & \text{if } x \text{ is a natural number} \\ x & \text{otherwise} \end{cases}$$


"HILBERT'S HOTEL"
©2002 CAROL LAY
THE INFINITE HOTEL WAS ALWAYS FILLED TO CAPACITY.
HILBERT'S HOTEL
NO VACANCY
WWW.WAYLAY.COM
YET, IF A NEW GUEST ARRIVED, SHE WAS ALWAYS GIVEN A ROOM.
AFTER ALL, THERE WERE AN INFINITE NUMBER OF ROOMS.
ROOM 84372279467141392645 91365365986527732 NEEDS FRESH TOWELS.

Infinite sets
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Thinking about Infinity

Prime Numbers

Proof by contradiction

Countable Infinite Sets

An infinite set is said to be *countable* if it is equivalent to the natural numbers.

Example:

the integers

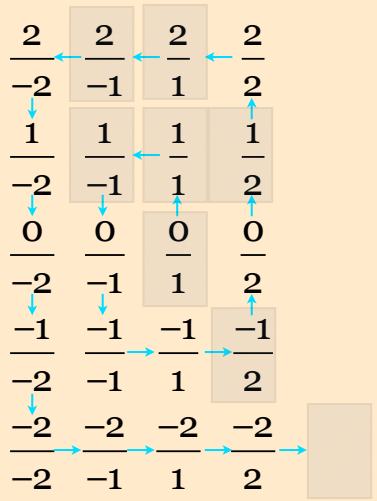
{0, 1, -1, 2, -2, 3, -3, ...}

Countable Infinite Sets

An infinite set is said to be *countable* if it is equivalent to the natural numbers.

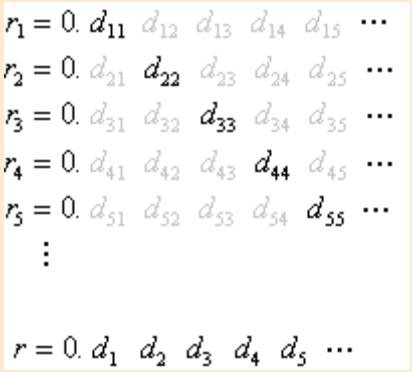
Example:

the rationals



"The Power of the Continuum"

The set of real numbers in the interval [0, 1] is not countable



proof by contradiction
[0, 1] is equivalent to whole real line, and even to the whole plane

The Devil's Challenge

Raymond Smullyan

Infinite sets
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The Strong Law of Small Numbers

#5. 40 / #6. 127 / #7. 432 / #8. 5777 / #9. not known

Infinite sets

Proof

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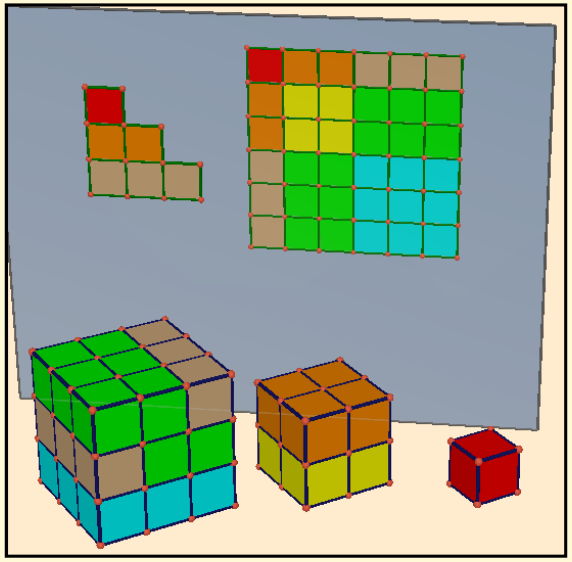
It seems like a power of 4 minus 1 is always a multiple of 3

	n	k	kover3
=	caseIndex − 1	$4^n - 1$	$\frac{k}{3}$
1	0	0	0
2	1	3	1
3	2	15	5
4	3	63	21
5	4	255	85
6	5	1023	341
7	6	4095	1365
8	7	16383	5461
9	8	65535	21845

Will this pattern break down?

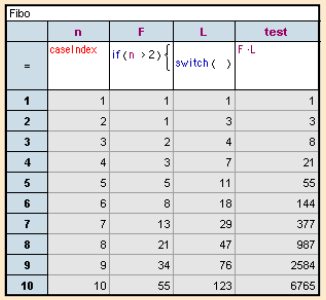
conjecture supplied by me

Generating conjectures



conjecture hinted at by me

Fibonacci conjectures



Fibo

	n	F	L	test
=	caseIndex	if(n > 2){	switch( )	F ·L
1	1	1	1	1
2	2	1	3	3
3	3	2	4	8
4	4	3	7	21
5	5	5	11	55
6	6	8	18	144
7	7	13	29	377
8	8	21	47	987
9	9	34	76	2584
10	10	55	123	6765

- start with student-generated conjectures, then make suggestions
- proofs by mathematical induction, algebraic manipulation, a method involving dominoes, and...

An explicit formula for Fibonacci numbers?

$$\left\lceil e^{\frac{n-1}{2}} \right\rceil$$

breaks down at 11
Actual formula: see work sheet on my Web site

Infinite sets
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Iterating Functions

Iterating Linear Functions

- In Algebra 2, an engaging introduction to sequences, series, and limits
- In this class, a prelude to the study of iterating non-linear functions, dynamical systems, and chaos

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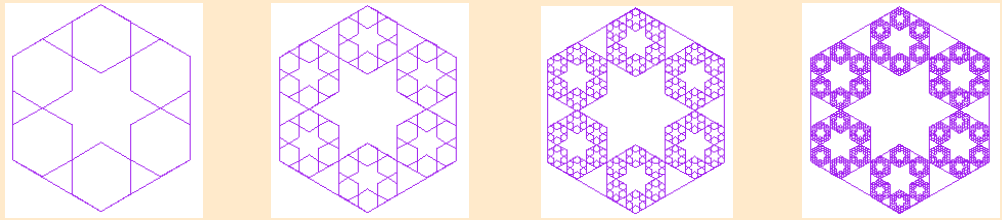

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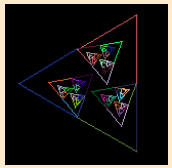

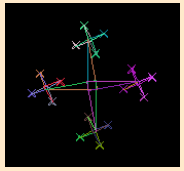

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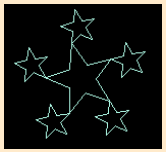
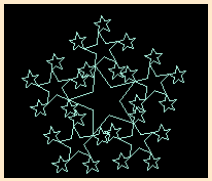
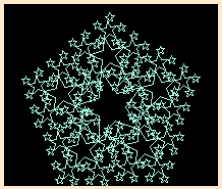
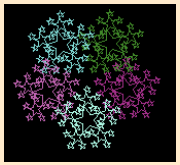

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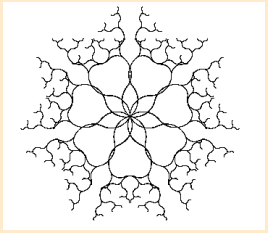

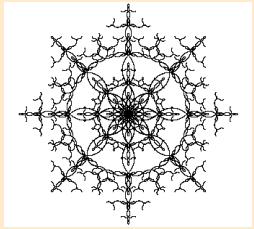

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