Instant Riches

Amazing Opportunity!

The following ad appeared in the school paper:

> Amazing investment opportunity at Algebank! Double your money instantly! Invest any amount! No amount is too small. Our bank will *double* the amount of money in your account every month. Watch your money grow!
>
> A service charge of $100 will be deducted from your account at the end of every month.

1. **Exploration.** Do you think this is a good deal? Why or why not? Use some calculations to back up your opinion.

2. Reg was interested in this investment. After calling to make sure that the $100 fee would be deducted *after* his money was doubled, he decided to join. However, after his service charge was deducted at the end of the fourth month, he discovered that his bank balance was exactly $0! How much money did he start out with? Explain.

Four other students invested their money. Gabe started with $45, Earl with $60, and Lara with $200. The figure shows a way to keep track of what happened to Lara's investment:

Month: 0 ... 1

$$200 \xrightarrow[\cdot 2]{} 400 \xrightarrow[-100]{} 300 \xrightarrow[\cdot 2]{} 600\ldots$$

3. a. Use arrows in this way to show what happened to each of their investments for the first five months.
   b. Give advice to each of these students.

Running Out of Money

4. Bea joined the plan, but discovered after one month that she had an account balance of exactly $0. How much money had she invested?

5. Lea discovered that she had an account balance of exactly $0 after two months. What was her initial investment?

6. Rea had an account balance of exactly $0 after three months. How much money did she start out with?

7. Summarize your answers to the previous three problems by making a table like the one below. Then extend the table to show up to at least ten months.

Months to Reach a Zero Dollar Balance

months	amount invested
1	
2	
…	

8. Describe the pattern in your table.

Gaining and Losing

9. Mr. Lear joined the plan, but discovered that at the end of every month he had exactly the same amount of money as when he started. How much money is it? Explain.

10. Algebank sends its customers statements quarterly (every three months). Several students were comparing their statements at the end of the first quarter. One had $50, another had $100, and a third had $150 in the account.
    a. What will happen to each one? Will all of them eventually gain money? What will their next quarterly statements look like? Explain.
    b. Explain how you can figure out how much money each of them started out with.

11.* Find two initial investment amounts that differ by $1, such that one of them will make money in this plan, and the other will lose money. How far apart will they be in 6 months? Explain.

12. **Report.** You have been asked to write an article on Algebank's investment plan for the Consumers' Guide column in the school paper. Write an article giving general advice to people wanting to join this plan. Describe the plan clearly and explain the pros and cons of joining it. Who will benefit from the plan? Who will lose in the long run? Explain, giving some examples. Make your article interesting, eye-catching, and readable.

13. **Generalization.** Use what you have learned in this lesson answer the following questions about plans with similar policies, but different numbers.
    a. Give advice to people wanting to join a plan, if their money is *tripled* every month, and the service charge is $100.
    b. Give advice to people wanting to join a plan if their money is doubled every month, but the service charge is $200.

14. * Suppose Algebank were to deduct the service charge *before* doubling the money. How would this change your answers to problems 12 and 13?

15. Describe another possible investment scheme and give advice to people about who should join and who should not.

More Banking

Reg works for Algebank. Last week, he was trying to analyze the investment plan described in "**Instant Riches**". Since he had studied algebra, he decided to uses x's and y's in his analysis. He wrote:

Let x = the amount of money the person invests

Let y = the amount of money the person has after one month.

Since the bank doubles the investor's money and deducts and \$100 fee, the function relating x and y is

$$y = 2x - 100$$

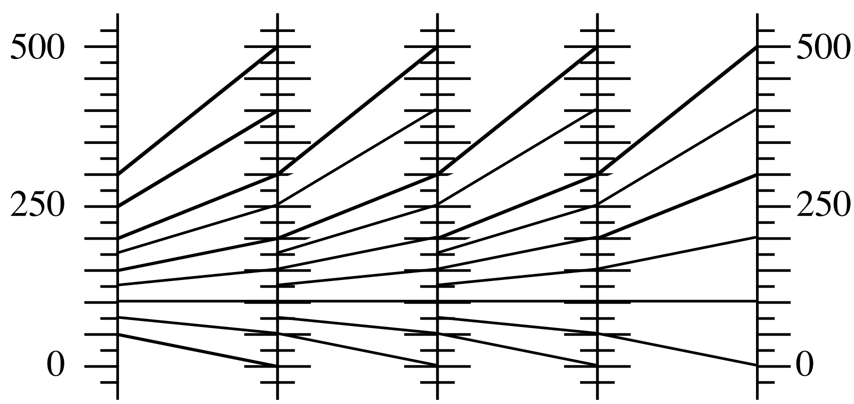
1. Make a function diagram for this function.

2. Use your function diagram to find out
   a. how much an investor started with who had \$300 after one month.
   b. how much an investor who started with \$300 had after one month.

3. Use your function diagram to find the amount of money the investor started with who ended up with the same amount of money after one month. (This is called the "fixed point" of the function.)

4. What happens to an investor who starts out with an amount of money less than the fixed point? What happens to an investor who starts out with an amount of money greater than the fixed point?

To analyze what happens to an investment over a period of more than one month, Reg had the idea of connecting function diagrams. Since the amount at the end of the first month is the amount at the beginning of the second month, he used the y number line from the first diagram as the x number line of the next, doing this as many times as he had room for on his paper.



5. Describe what the linked function diagrams show.

6. How could one use a single function diagram to follow what would happen to an investment over a period of more than one month?

7. € Use Reg's method to analyze a plan where the investment is multiplied by 1.5 and the service charge is \$50. Describe what your linked diagrams show.

8. Compare this plan with the previous one for someone who invests:
   a. $90   b. $100   c. $110

9. Which do you think has a bigger influence on the amount of money the investor makes, the service charge, or the number by which the investment is multiplied? Write an explanation supporting your opinion. Use several examples.

10. Explain why Al thought it was important to know whether the service charge was deducted before or after the money was doubled. Use some examples. Express each policy with a function.

11. **Report.** Write a report on investment plans of the type studied in this assignment and in Lesson 1, plus, optionally, other plans of your design. Use variables. Your report should include, but not be limited to, answers to the last two problems.

12. **Project.** Find out what the service charge and interest rate is at three real banks. Figure out what would happen to $100 invested at each one over a period of three years. Write up what you discover as if it were an article for the school newspaper, and you were giving advice to students on where to put their savings.

Iterating Linear Functions

You will need: graph paper or function diagram paper

Tree Harvesting

Paul's Forestry Products own two stands of trees. This year, there are about 4500 trees in Lean County, and 5500 in Cool County. So as to not run out of trees, their yearly harvesting policy at each location is to cut down 30% of the trees and then plant 1600 trees. For example, in Lean County this year, they will cut 1350 trees and plant 1600 trees.

1. Make a table of values showing how many trees they would have at each location every year for 9 years.

2. Describe the change in the number of trees at each location. Is it increasing or decreasing? Is it changing at a constant rate from year to year? What do you think will happen in the long run?

3. Write a formula that would give the number of trees next year in terms of the number of trees this year. (You may use y for next year's number, and x for this year's number. What you get is called a *recurrence equation*.)

4. How many trees would they have at each location after 30 years?

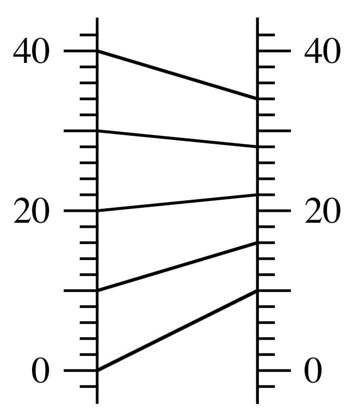
Drugs

To control a medical condition, Shine takes 10 milligrams of a certain drug once a day. Her body gets rid of 40% of the drug in a 24 -hour period. To find out how much of the drug she ends up with over the long run, we can use function diagrams.

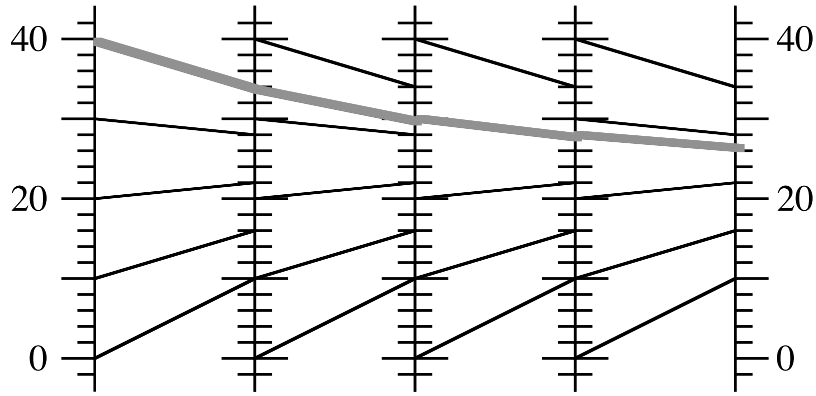
5. Explain why the recurrence equation is $y = .6x + 10$

6. Make a table of values for the recurrence equation, using these values for x: 0, 5, 10, 15, 20, 25, 30, 35, 40. (This only represents a single day.)

Here is a function diagram for the recurrence equation:



This function diagram can be repeated to show what happens over the long run. The linked diagrams show how the y-values for one become the x-values for the next.



7. Use the diagram to predict what happens in the long run if Shine takes 10 mg a day of the drug after an initial dose of:
   a. 10 mg
   b. 25 mg
   c. 40 mg

8. Check your predictions by calculation.

Remember that instead of linked diagrams as in the figure above, one could use a single function diagram of the function: just follow an in-out line, then move horizontally across back to the x number line; then repeat the process, using the in-out line that starts at that point.

Savings

Glinda puts $50 a month into a savings account with yearly interest 6%, compounded monthly.

9. What is the interest per month?

10. How much money will she have at the end of one year?

11. Write a recurrence equation for this problem, expressing the amount in the account at the end of each month.

12. Make a function diagram.

13. € How does what happens in the long run for this problem differ from the problems in the previous sections? Explain.

Definition: To *iterate* a function means to use its output as a new input.
All the problems in this lesson involve iterating linear functions. We will use function diagrams and algebraic symbols to get a more general understanding of this kind of problem.

14. € Describe the difference between function diagrams for $y = mx + b$ when:
    a. $0 < m < 1$
    b. $m = 1$
    c. $m > 1$

Analyzing the Sequences

Definition: a fixed point of a function is one in which the output is the same as the input.

Example: For the function $y = 7x - 12$, when the input is 2, the output is also 2.

1. Find the fixed points:
   a. $y = 3x - 6$ b. $y = 3x + 5$
   c. $y = 3x$ d. $y = x$
   e. $y = x + 3$ f. * $y = x^2 - 2$

2. € Function diagrams may help you think about these questions:
   a. There is a linear function that has more than one fixed point. What is it? Explain.
   b. What linear functions have no fixed points? Explain.

3. **Generalization.**
   a. Find a formula for the fixed point for the function $y = mx + b$. (Hint: since the output is the same as the input, substitute x for y, and solve for x.)
   b. Explain why $m = 1$ is not acceptable in the formula you found. What does that mean in terms of the existence of the fixed point for equations of the form $y = x + b$?

When iterating a function, you get a sequence of numbers.

4. **Exploration.** Start with the equation $y = 2x + 3$. Change one number in the equation so that when iterating the function, starting with any input, you get:
   a. an arithmetic sequence
   b. a geometric sequence
   c. a sequence where the values get closer and closer to a fixed point
   Compare your answers with other students.

5. **Generalization.** When iterating $y = mx + b$, different things may happen, depending on the value of the parameters. Find the values of m and b which lead to the following situations:
   a. arithmetic sequences
   b. geometric sequences
   c. sequences where the values get further and further from the fixed point
   d. sequences where the values get closer and closer to the fixed point

6. **Report.** Summarize what you know about iterating linear functions. Include, but do not limit yourself to:
   - real world applications
   - use of function diagrams
   - the fixed point
   - these special cases:
     ◊ $b = 0$
     ◊ $0 < m < 1$
     ◊ $m = 1$
     ◊ $m > 1$

Asthma

For her asthma, Lynne takes 360 mg of the drug theophylline twice a day. After 12 hours, 60% of the drug has been eliminated from the body .

1. Assume she has x_a mg of the drug in her body immediately after taking the dose. Explain why $y_a = .4x_a + 360$ is the recurrence equation that says how much will be in her body immediately after taking the next dose.

2. Assume she has x mg of the drug in her body immediately before taking the dose. Explain why $y_b = .4\ (x_b + 360)$ is the recurrence equation that says how much will be in her body immediately before taking the next dose.

The amount of theophylline in her body is constantly changing, but the lowest amount (right before taking the drug),and the highest amount (right after), eventually approach a stable level.

3. Find that level, using tables or function diagrams of the recurrence equations. What is the level before taking the dose? what is it after?