# $n^{th}$ Power Variation

This is a unit I used to teach to 9th and 10th graders in a Math 2 course before my retirement, slightly edited. It is part of a suite of lessons on recognizing functions from *x*-*y* tables. I presented a rationale for the importance of this work in this blog post.

In spite of its importance in both science and math, $n^{th}$ power variation ($y = kx^n$) is inexplicably missing from the Common Core State Standards for Math. This is unfortunate. In addition to applications, a discussion of these functions helps justify fractional and negative exponents, a topic that is notoriously difficult to motivate.

## Overview

It is not exactly a prerequisite, but in preparation for this unit, it would be useful to use some examples in a discussion of the *add-add* pattern for linear functions: when you add $c$ to $x$, you add $mc$ to $y$. Naturally, make the connection to slope.

Depending on how things are sequenced in your program, you may also want to discuss *add-multiply* on examples of exponential functions. (When you add $c$ to $x$, you multiply $y$ by $b^c$.)

This unit is not comprehensive, as it is a snapshot from a constantly evolving school-based curriculum, and thus it makes some assumptions about what came before and after. Still, you may find it useful as you think about teaching these and related topics:

◊ $n^{th}$ power direct and inverse variation
◊ recognizing functions from data points (mathematical modeling)
◊ scaling perimeter, area, and volume
◊ square root and cube root functions
◊ fractional and negative exponents

Note that asking students for an algebraic proof of the *multiply-multiply* pattern is not attempted until page 8. The unit offers opportunities for earlier discussions, but if you want broad understanding, any such discussion must be based on students having experienced the pattern with actual numbers first.

The connection with fractional and negative exponents is made, but the unit does not really include much work on that. Fortunately, it should not be difficult to find material on that topic in standard textbooks.

## Scaling Tables (p. 1)

Whenever possible, it is a good idea to build algebraic work on a visual and geometric foundation.

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#3-4 require physical cubes, preferably the kind that interlock in all directions. Those are readily available from math education publishers.

There is a similar activity in *Geometry Labs* 10.5.

## *n*th Power Variation (p. 2)

Note that we wait until the students have some experience with what we are talking about before introducing the terminology.

#3 introduces *multiply-multiply,* and deserves a class discussion to summarize what was learned.

For #4 the idea is *not* to graph individual points and connect them. Students are to find the formula, with the help of what they learned in #3 and some algebraic manipulation. That will allow them to make the graph on an electronic grapher. The reason for sketching the graphs is to start to develop a visual sense for these functions.

## Recognizing *n*th Power Variation (p. 3)

#1-4 are about applying *multiply-multiply.*

Most students are likely to answer #2 by looking at examples. If any offer an algebraic proof, perhaps they can be asked to share it with the class. If not, use your judgment about whether to offer such a proof yourself. We are not yet halfway through the unit, so there will be time for that.

Note that #3a is linear, and #3c is exponential.

#5-6 introduce fractional exponents. Unless your students are already familiar with fractional exponents, and even if they are, a discussion is in order before tackling #5. Because this is difficult for many students, it is probably sufficient to use the laws of exponents to explore the exponents ½ and ⅓.

Again, for #6, the students should graph electronically, based on the formulas they discovered, not individual points. It is fine, of course, to also display those points.

## A New Meaning for Exponents (pp. 4-5)

#4-8 offer a justification of the rules for manipulating radicals, and #10 is useful to make the rules concrete by using familiar numbers.

*Geometry Labs* 9.3-9.4 (and some of the preceding labs) offer a geometric justification for the rules governing the manipulation of radicals.

*Algebra: Themes, Tools, Concepts* (mathed.page/attc/) Lessons 9.5-9.9 approach the exponent ½ from a "functions" point of view, and can provide additional material on this.

## Find the Formula (pp. 6-7)

This applies the previous lessons to "real world" problems. Depending on your policies, those can be assigned as homework, or saved for later review.

## $n^{th}$ Power Variation Graphs and Tables (p. 8)

This is a big-picture summary of many key ideas in this unit. #2 asks for an algebraic proof of *multiply-multiply*.

## Inverse $n^{th}$ Power Variation (pp. 9-10)

This can be seen as completing the picture, or if this unit is already too much for your class, it can be seen as an extension. The connection is made with negative exponents.

One particular case of inverse variation is the subject of the Perspective Lab (mathed.page/alg-2/perspective), which can be carried out without this introduction, by using the *constant sums, constant products* approach. (mathed.page/constant)