Angles Teacher Notes

Introduction

This unit is mostly intended to strengthen students' understanding of what an angle is. A surprising number of students get to high school without a strong sense of this, and therefore are seriously handicapped in geometry.

The unit can also be used to reinforce number sense and mental calculations skills. Indeed, students should calculate the answers, not measure. In most future math classes, angles are more often calculated than measured.

If you want to use the unit to teach students how to use a protractor, you might wait until later in the unit when they know what they are measuring and have a sense of what sort of numbers to expect.

The last two lessons involve reasoning and calculations about angles, and depend on having the grounding provided by the start of the unit.

How to organize these lessons

These lessons lend themselves to student collaboration. Students can work individually, and then share and compare their answers with neighbors. Or they can work in pairs, or groups of four. In any case, the lessons should be interactive, with the teacher helping individuals and the whole class as needed.

Angles Around a Point, Angle Measurement

These lessons are from *Geometry Labs*. You can find teacher notes and solutions at
http://www.mathedpage.org/geometry-labs/
They are intended to be engaging, have next to no prerequisites, and involve a lot of reasoning. They lay the foundation for the rest of the unit.

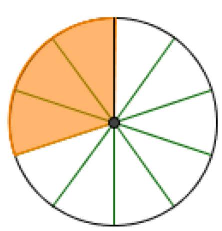
"Pizza slices"

The next worksheets are all based on the same images, so student familiarity with angles will grow as they work through them. Do not expect any of the worksheets to take a full class period.

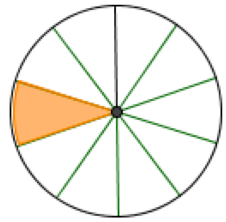
Encourage students to use colored pens or pencils to make the math as clear as possible.

The Smallest Angles

In each circle, there are many angles, each consisting of one or more "pizza slices". In future lessons, students will work with angles larger than the smallest one, such as this one:



In this lesson, we just consider the smallest ones in each circle, the ones consisting of a single pizza slice, such as this one:



Find the Angles

Starting with this lesson, students may use angles made of multiple slices.

Supplementary Angles, Complementary Angles

Encourage students to find examples different from their neighbors, and to find as many different answers as possible.

Let the students discover that they cannot carry out the assignment in every circle. When a student asks about it, lead a class discussion about what is needed for the assignment to be possible.

Vertical Angles

Again, you cannot find vertical angles in every circle. Lead a discussion of what is required for a circle to exhibit vertical angles.

Again, encourage students to find as many different-sized angles as they can.

Allow students to find multiple pairs in one circle. If the circles are completely colored in several colors, the results are aesthetically pleasing. (This is a particularly straightforward lesson, which could be finished as homework once students get the basic idea.)

Angles in a Triangle

This lesson is from *Geometry Labs*. You can find teacher notes and solutions at
http://www.mathedpage.org/geometry-labs/

If students have never seen the definitions of "equilateral", "isosceles", and "scalene", you of course have to explain the meanings of those words. If, on the other hand, they have seen them, do not start with a review: the activity itself will provide a review, and students should figure out the meaning of "isosceles right triangle" and the like. As they work, you might interrupt them for a class discussion if you see widespread misconceptions. If on the other hand most students are on it, just talk individually to the ones who have trouble putting those concepts together.

Triangles in a Circle

This lesson is adapted from *Geometry Labs*, Lab 1.7. You can find teacher notes and solutions at
http://www.mathedpage.org/geometry-labs/

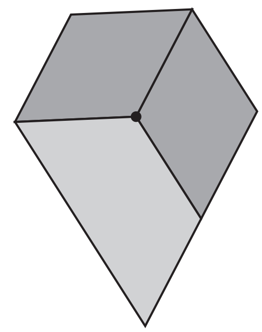
LAB 1.1

Name(s) ____________________

Angles Around a Point

■ **Equipment:** Pattern blocks

Place pattern blocks around a point so that a vertex (corner) of each block touches the point and no space is left between the blocks. The angles around the point should add up to exactly 360°.



For example, with two colors and three blocks you can make the figure at right.

Use the chart below to keep track of your findings.

- Every time you find a new combination, circle the appropriate number on the list below.
- Cross out any number you know is impossible.
- If you find a possible number that is not on the list, add it.

Since the two-colors, three-blocks solution is shown above, it is circled for you.

Colors:	**How many blocks you used:**								
all blue	3	4	5	6						
all green	3	4	5	6						
all orange	3	4	5	6						
all red	3	4	5	6						
all tan	3	4	5	6						
all yellow	3	4	5	6						
two colors	3	4	5	6	7	8	9	10	11	12
three colors	3	4	5	6	7	8	9	10	11	12
four colors	3	4	5	6	7	8	9	10	11	12
five colors	3	4	5	6	7	8	9	10	11	12
six colors	3	4	5	6	7	8	9	10	11	12

How many solutions are there altogether? __________

Discussion

A. Which blocks offer only a unique solution? Why?

B. Why are the tan block solutions only multiples of 4?

C. Explain why the blue and red blocks are interchangeable for the purposes of this activity.

D. Describe any systematic ways you came up with to fill in the bottom half of the chart.

E. How do you know that you have found every possible solution?

F. Which two- and three-color puzzles are impossible, and why?

G. Which four-color puzzles are impossible, and why?

H. Why is the five-color, eight-block puzzle impossible?

I. Which six-color puzzles are impossible, and why?

LAB 1.2
Angle Measurement

Name(s) ____________________

■ **Equipment:** Pattern blocks, template

1. What are the measures of the angles that share a vertex at the center of

a. A Chrysler symbol? ___________

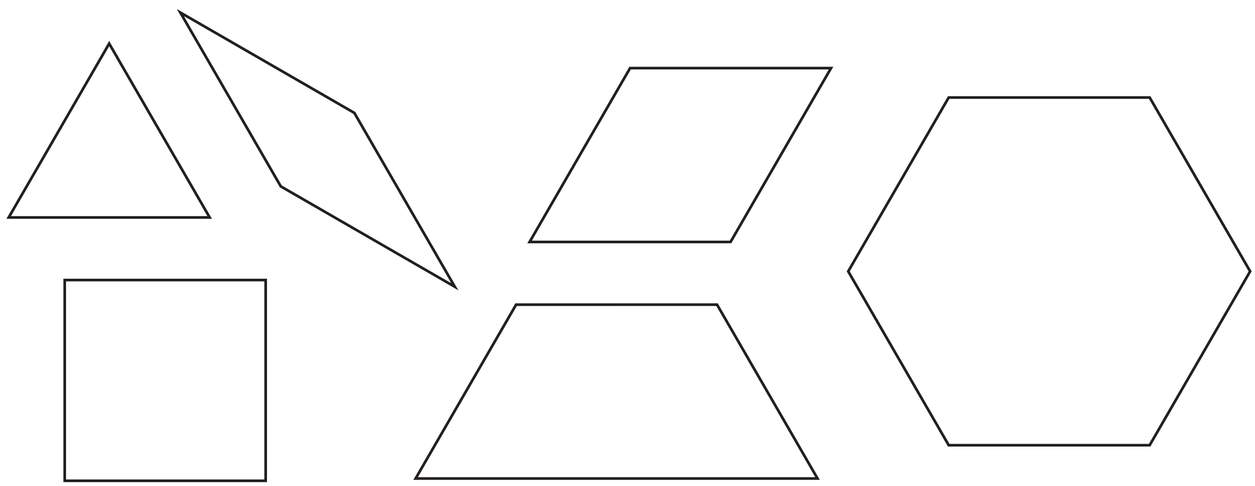
b. A Mercedes symbol? ___________

c. A peace sign? ___________

d. A clock, between consecutive hours? ___________

e. A cross? ___________

2. Find the measures of all the angles for each of the pattern blocks shown below. Write the angle measures in these shapes.

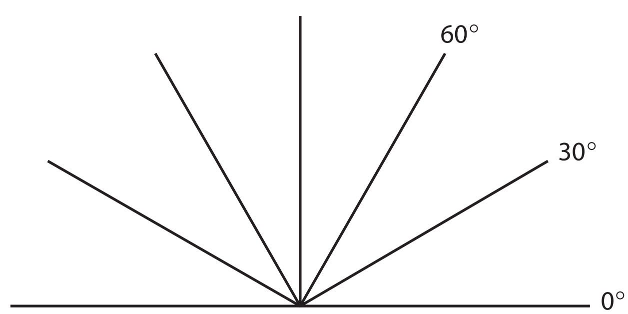


3. One way to measure angles is to place smaller angles inside larger ones. For example, six copies of the small angle on the tan pattern block fit inside the figure below. This figure, called a **protractor**, can be used to measure all pattern block angles.

a. Mark the rest of the lines in the figure with numbers.

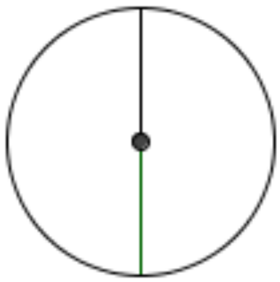
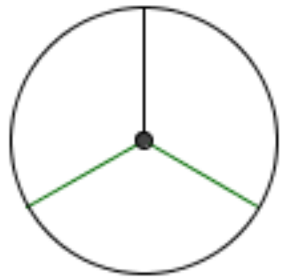
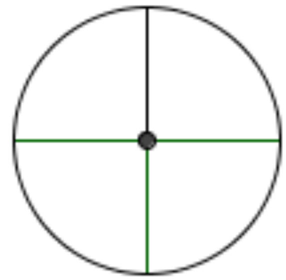
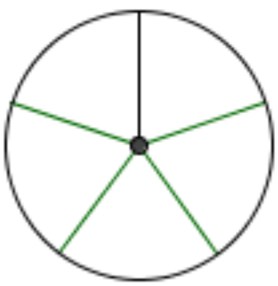
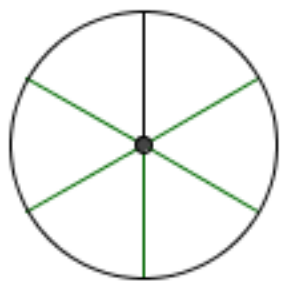
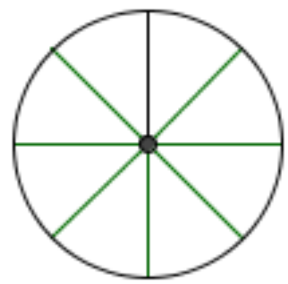
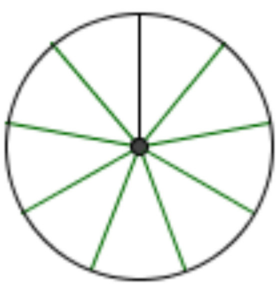
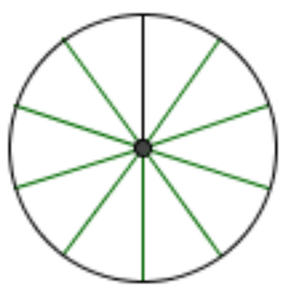
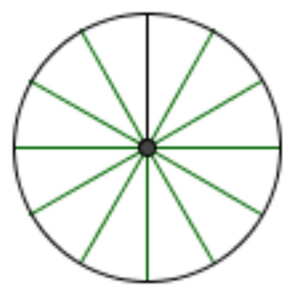
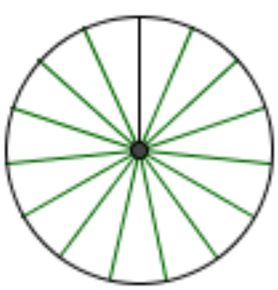
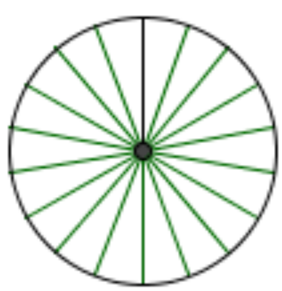
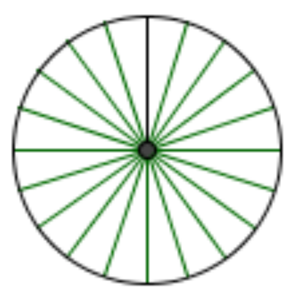
b. Use it to check the measurements of the pattern block angles.

c. Using the tan pattern block, add the 15° lines between the 30° lines shown on the protractor.



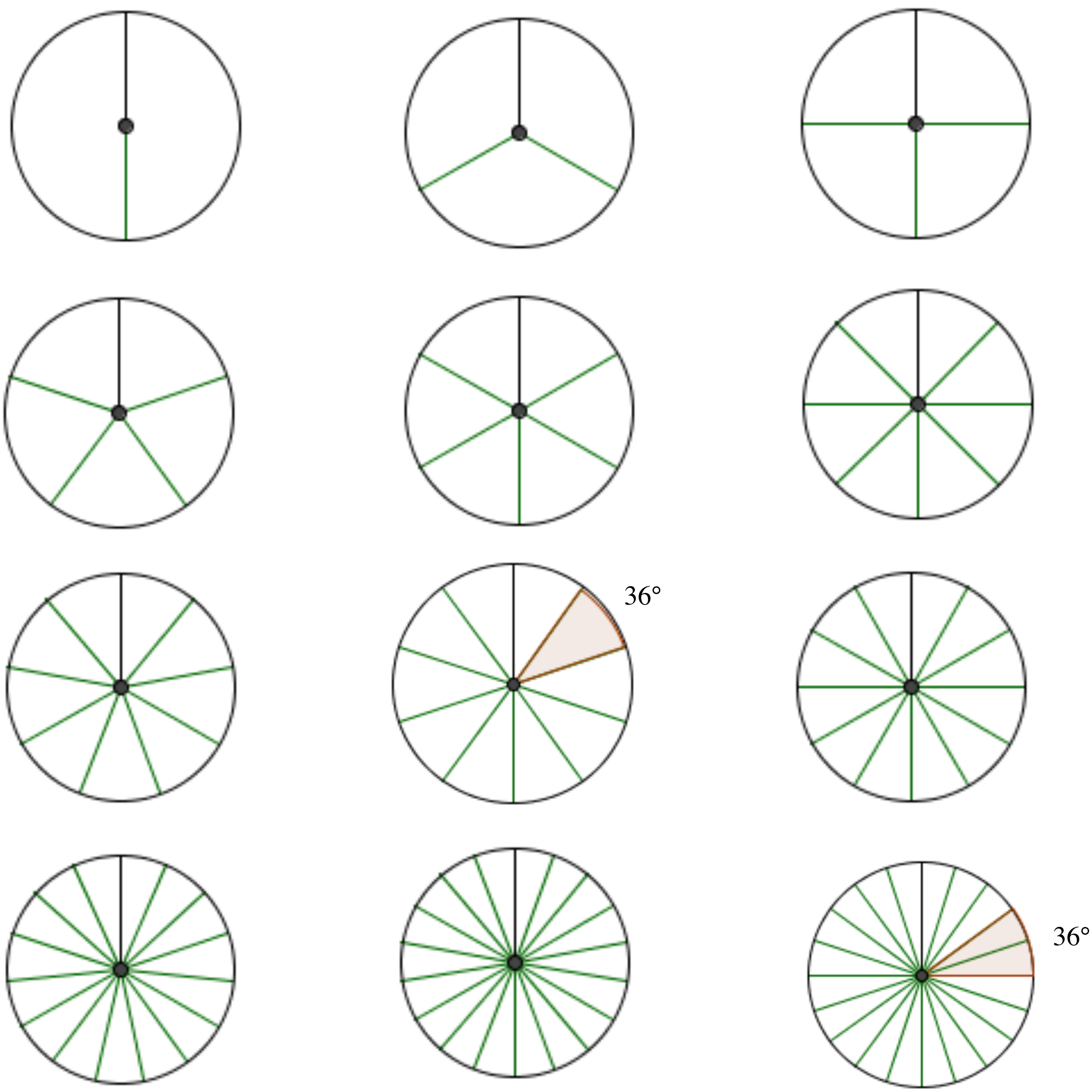
The Smallest Angles

There are 360° around a point. Each circle below is divided into equal sections.
For each circle, calculate the measure of the smallest angle. Write it next to the circle.

Find the Angles

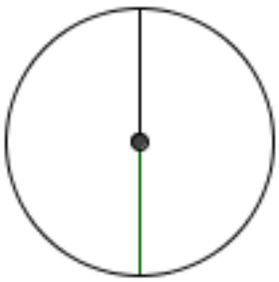
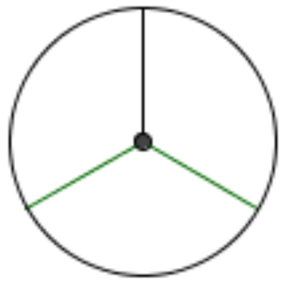
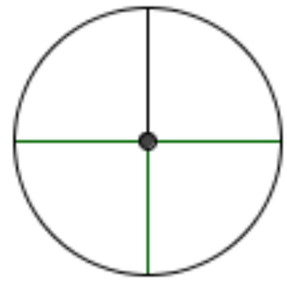
1. Find and shade in angles with each given measure in as many circles as possible. Write the corresponding number near the angle. The first one has been done for you.
   a. 36°
   b. 60°
   c. 72°
   d. 90°
   e. 180°

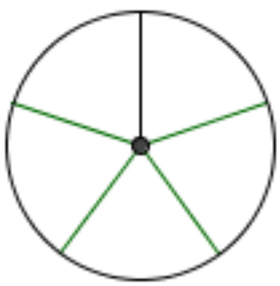
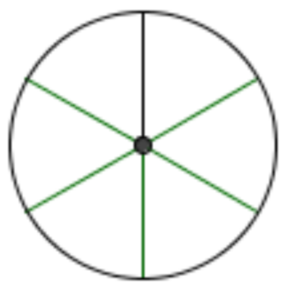
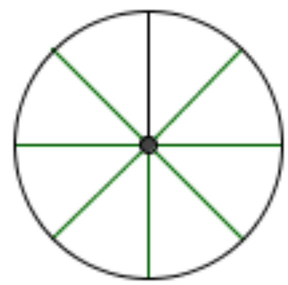


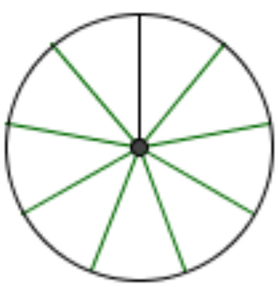
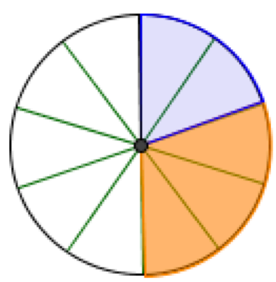
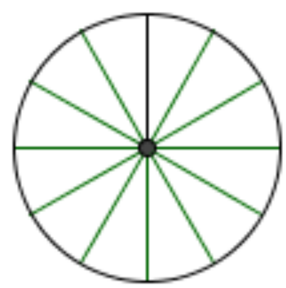
2. Choose an angle and challenge a classmate to find it, only giving them the size of the angle.

Supplementary Angles

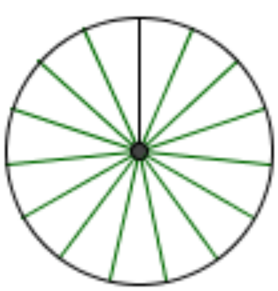
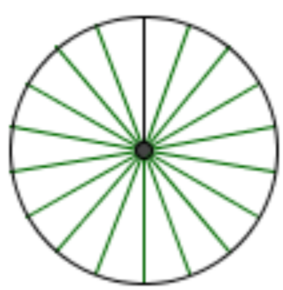
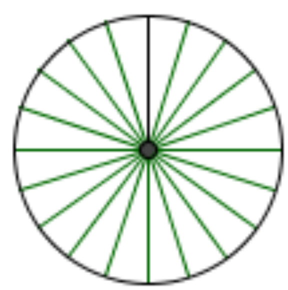
In this lesson, you are to identify pairs of angles that add up to 180°.
Shade each pair of angles in using two colors, and write the addition next to the circle.
Try to find examples that are different from your neighbors'.
One has been done for you.

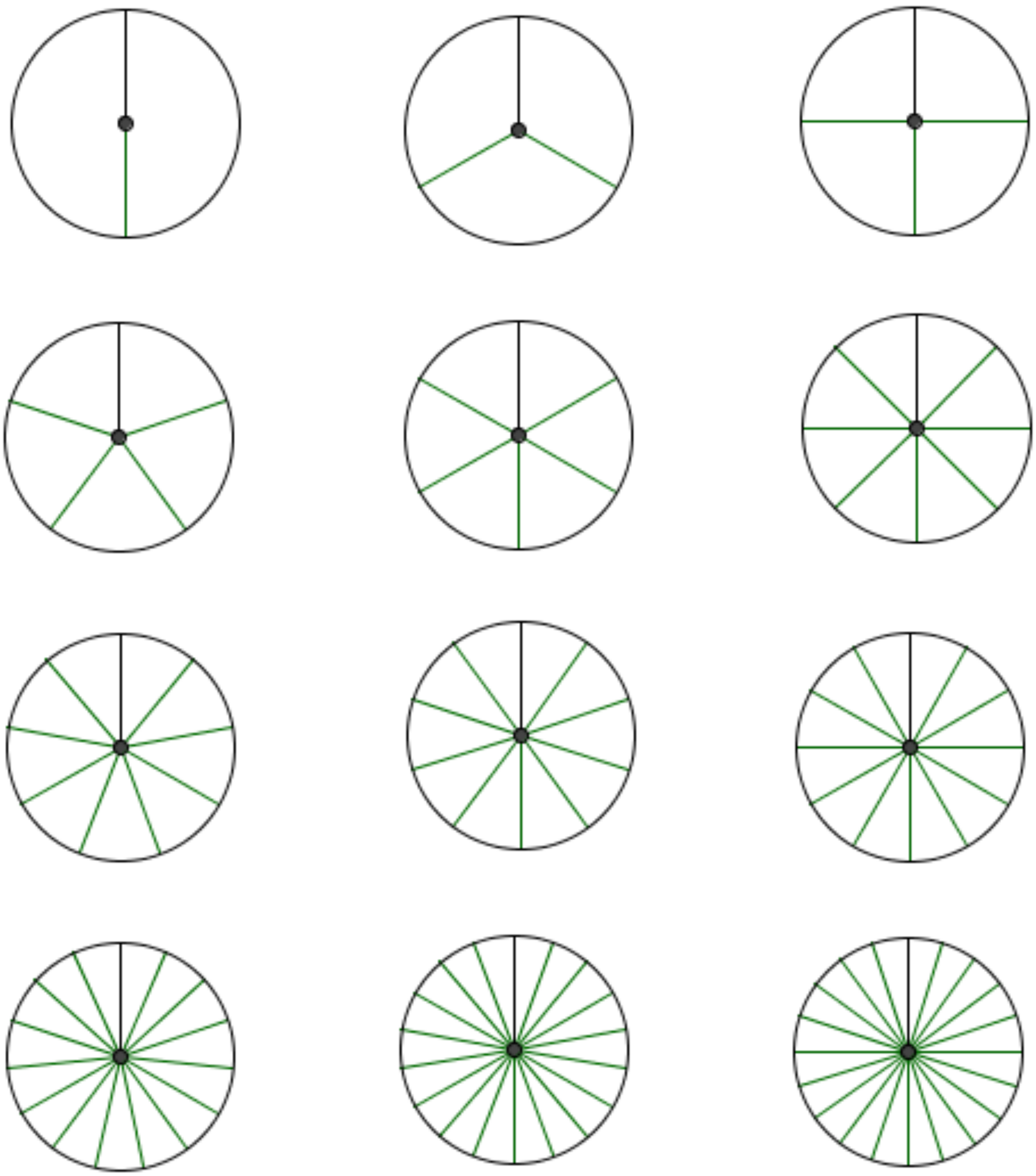




72°+108°=180°

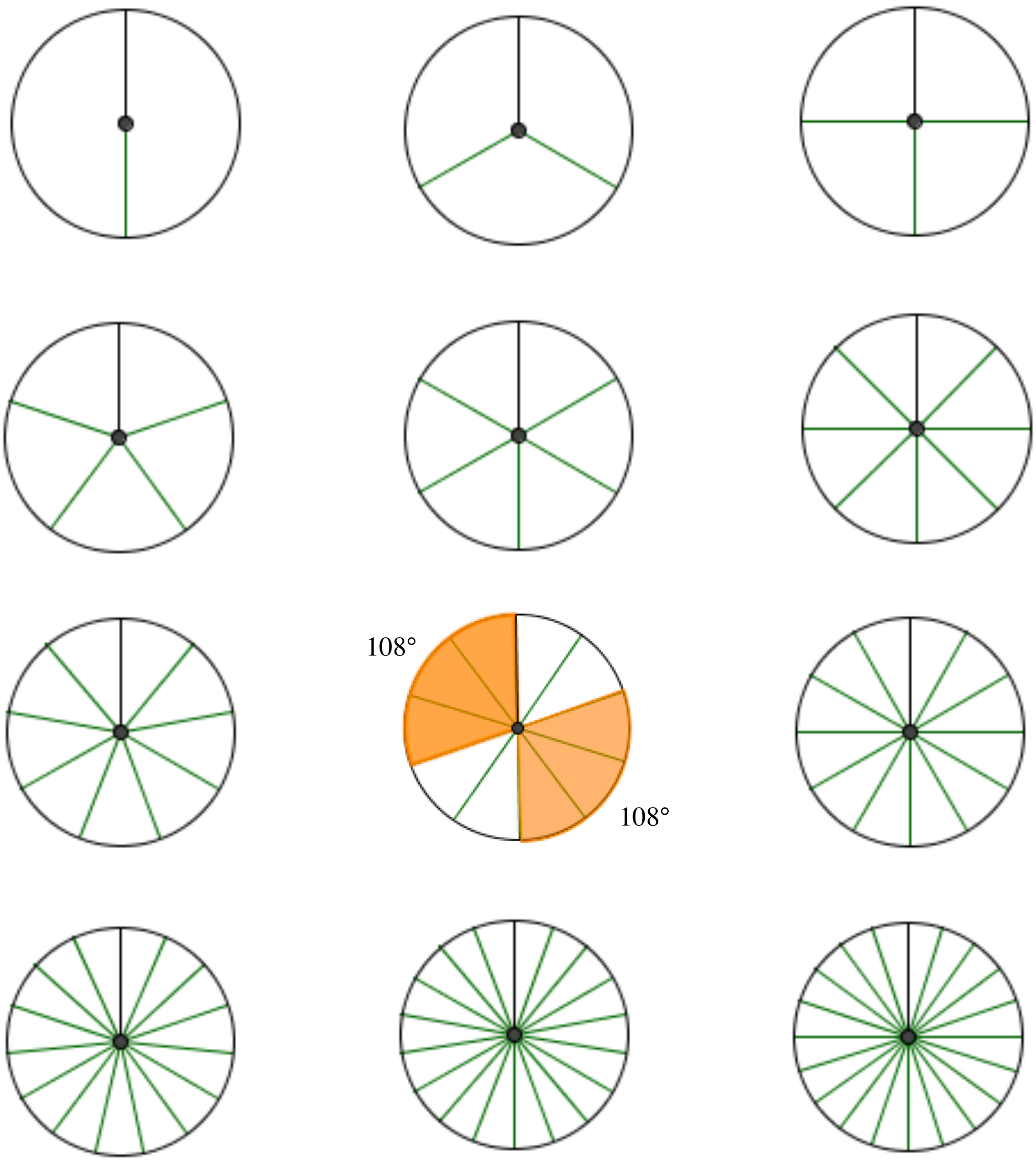
Complementary Angles

In this lesson, you are to identify pairs of angles that add up to 90°.
Shade each pair of angles in using two colors, and write the sum next to the circle.
Try to find examples that are different from your neighbors'.



Vertical Angles

Shade in pairs of vertical angles, and write their measurements next to them.
Try to find examples that are different from your neighbors.
One has been done for you.



LAB 1.5
Angles in a Triangle

Name(s) ____________________

- **Equipment:** Template

Types of triangles

Obtuse: contains one obtuse angle
Right: contains one right angle
Acute: all angles are acute

1. Could you have a triangle with two right angles? With two obtuse angles? Explain.

2. For each type of triangle listed below, give two possible sets of three angles. (In some cases, there is only one possibility.)
   - **a.** Equilateral: ______________, ______________
   - **b.** Acute isosceles: ______________, ______________
   - **c.** Right isosceles: ______________, ______________
   - **d.** Obtuse isosceles: ______________, ______________
   - **e.** Acute scalene: ______________, ______________
   - **f.** Right scalene: ______________, ______________
   - **g.** Obtuse scalene: ______________, ______________

3. If you cut an equilateral triangle exactly in half, into two triangles, what are the angles of the "half-equilateral" triangles? ______________

4. Which triangle could be called "half-square"? ______________

5. Explain why the following triangles are impossible.
   - **a.** Right equilateral
   - **b.** Obtuse equilateral

Name(s) ____________________

6. Among the triangles listed in Problem 2, which have a pair of angles that add up to 90°?

7. Make up two more examples of triangles in which two of the angles add up to 90°. For each example, give the measures of all three angles.

____________, ____________

8. Complete the sentence:
"In a right triangle, the two acute angles . . . "

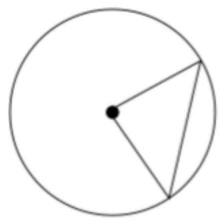
9. Trace all the triangles on the template in the space below and label them by type: equilateral (EQ), acute isosceles (AI), right isosceles (RI), obtuse isosceles (OI), acute scalene (AS), right scalene (RS), half-equilateral (HE), obtuse scalene (OS).

Triangles in a Circle

Equipment: circle geoboard, circle geoboard paper

Types of triangles:

Equilateral (EQ)	Acute Isosceles (AI)
Right Isosceles (RI)	Obtuse Isosceles (OI)
Acute Scalene (AS)	Right Scalene (RS)
Half-Equilateral (HE)	Obtuse Scalene (OS)

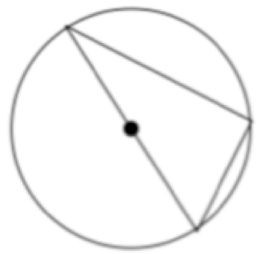


1. Make triangles on the circle geoboard, *with one vertex at the center,* and the other two on the circle, as in the figure above.
   a. Make one of each of the eight types listed above, if possible.
   You do not have to do them in order!
   b. Sketch one of each of the types of triangles it was possible to make on circle geoboard paper. Identify which type of triangle it is, and label all three of its angles with their measures in degrees. (Do not use a protractor! Use logic and calculations to figure out the angles.)

2. Thinking back:
   a. What is true of all the possible triangles in #1? Explain.

   b. Summarize your strategy for finding the angles. Describe any shortcuts you used.

Definition: A triangle is *inscribed* in a circle if all three of its vertices are on the circle.



3. Make triangles on the circle geoboard, with inscribed triangles such that the circle's center is *on a side* of the triangle as in the figure above.
   a. Make one of each of the eight types listed above, if possible.
      You do not have to do them in order!
   b. Sketch one of each of the types of triangles it was possible to make on circle geoboard paper. Identify which type of triangle it is, and label all three of its angles with their measures in degrees. (Do not use a protractor! Use logic and calculations to figure out the angles.)
      Hint: drawing an additional radius should help you find the measures of the angles.

4. Thinking back:
   a. What is true of all the possible triangles in #3?

   b. Summarize your strategy for finding the angles. Describe any shortcuts you used.