Teacher Notes for Functions Unit

Henri Picciotto
www.MathEducationPage.org
henri@MathEducationPage.org

General Notes

I have taught most of the lessons in this unit in various contexts, so I am confident that they "work". This document is based on my classroom experience, and I hope you will find it helpful.

I have mostly used these lessons with students organized in cooperative groups of four. Mostly, they worked independently, but they can ask for help from each other and from me when they need it. I don't usually start by telling students "how to do it", nor do I expect them to discover everything on their own. I find that the best strategy is to go back and forth between whole-class, teacher-led discussion, and students working individually and in small groups.

I group students randomly, and change groups every two weeks. I find that this avoids students trying to second-guess why I put them in a particular group, and as they get used to the frequent changes, they stop worrying about who they are grouped with. Another advantage of random groups is that sometimes you get homogeneous groups, and sometimes you get heterogeneous groups. There are advantages and shortcomings to both, and this way you are not stuck with one or the other.

If your students are not used to the guided-inquiry style of these lessons, you will need to support them more, but you should avoid doing all the work for them. The philosophy of this sort of curriculum is to engage the students' common sense, rather than their ability to memorize. It is always possible, and in fact necessary, for the teacher to help students along the way, and to put conclusions into words. But my experience is that students cannot hear me give answers to questions they don't have. The main point of the guided inquiry is to help them understand what the questions are, so that when I give an explanation, they can hear it.

I recommend you work through the lessons yourself before teaching them.

Sources

Most lessons are taken or adapted from two sources:

Algebra: Themes, Tools, Concepts, by Anita Wah and Henri Picciotto
Geometry Labs, by Henri Picciotto

Both books and corresponding teacher notes and solutions are available for free download on
www.MathEducationPage.org

Suggested Course

As you can see, most of the lessons address many standards. In the table of contents of the packet, I listed a suggested course for the lessons (7^{th} grade, 8^{th} grade, or Algebra 1), but you may want to make your own choices for your classes, based on your preference and what you think is most appropriate for your students. You may also consider teaching the lessons in a different order.

Investigating Rectangle Areas

This lesson is an opportunity to review proportional relationships from a functions point of view.

Do not start this lesson by teaching or reminding students about how to graph y=mx+b or y=mx. Instead, have them enter the points from the table into graphs, and connect them. Of course, in the long run that is not the most efficient way to make these graphs, but the point of this lesson is to show the relationship between points, coordinates, and formulas. A good time to talk about y = mx is at the end, after most students have done #8, and after a whole-class discussion of #8.

The "x by x" graph is very important. Its purpose is to remind students that not all graphs are straight lines, and that not all relationships are proportional.

The lesson is adapted from *Algebra: Themes, Tools, Concepts*, Lesson 1.A.
Teacher notes and some solutions are in the corresponding section of the Teacher's Guide.

Scaling on the Geoboard / Similar Rectangles

This lesson makes a connection between the scaling of figures, proportional relationships, equivalent fractions, fractions and decimals, and slope. (Slope is mentioned in the 7^{th} grade standards in the context of proportional relationships. In 8^{th} grade and beyond, it is part of the work on functions.)

There are teacher notes and solutions in *Geometry Labs*.

Guess My Function

This is an introduction to the vocabulary of functions, and to working in three different representations: formulas, tables, graphs.

The game "Guess My Function" can be played with the whole class with various functions as a warm-up almost any time. It can reinforce mental calculations (which are extremely important in the age of calculators), as well as many skills and concepts.

Discounts

This lesson is a comparison of a proportional relationship (the graph goes through the origin) and a non-proportional relationship. It shows that some problems can best be understood with the help of a graph. It also provides an opportunity to attach a meaning to the m and the b in y=mx+b.

Do not require students make the graph a certain way. Most will probably need to start by making a table of values. Others may figure out a formula and use their knowledge of y=mx+b to make the graphs.

Either way, the discussion at the end of the lesson should include these questions:

What is the independent variable in this problem?
What is the dependent variable in this problem?
What are the formulas
Which function is a proportional relationship?
Which function is linear?
What is the meaning of m in this problem?
What is the meaning of b in this problem?
How is the amount of money you save by buying the discount card (or not) visible on the graph?

It is not necessary for every student to solve #6.

This problem is adapted from *Algebra: Themes, Tools, Concepts* Lesson 6.A.

Constant Speed

This lesson makes the connection between distance-time graphs and proportional relationships.

#1 is a way to introduce or review the basic idea of distance-time graphs.

#3 is a chance to review proportional relationships. Important ideas need to be reviewed often, and as much as possible in different formats and contexts. Reviewing them in exactly the same way is boring for some students, and discouraging for others. Making connections between different formats and contexts helps access for some students who only get it after several different tries, and it also helps depth of understanding for others, forcing them to go beyond rote memorization.

#4 is an acknowledgment that problems in math class are often simplified from the "real world".

The lesson is adapted from *Algebra: Themes, Tools, Concepts*, 4.1.

In the Lab

This lesson is not unlike **Discounts**, as it is a comparison of a proportional relationship with another linear function. It too provides an opportunity to attach a meaning to the m and the b in y=mx+b.

#21 is a good one for a discussion of slope. It is not essential to have the right answer, as long as the discussion throws light on the meaning of density.

The lesson is adapted from *Algebra: Themes, Tools, Concepts*, 4.6. In that book, it is followed by another lesson in the same context, featuring measurements that are not as perfect.

Polyomino Perimeter and Area

This lesson offers an opportunity to compare a linear and a non-linear function. It also shows that you can have a function even if you don't know a formula for it, as long as there is a way to find the output for each input.

This is my favorite lesson, and I have presented it in grades 4-12 at different times, as well as to teachers and parents. It is a nice mix of very accessible and somewhat more difficult content. It is also a good opportunity to show the connections between algebra and geometry.

It can be found in *Geometry Labs,* where it is followed up with another lab on the same subject.

Weight as a Function of Age

This lesson is another one about a function without a formula. In addition, it is a way to look at average rate of change (slope) in the case of a non-linear function. It is based on real data.

It is part of *Algebra: Themes, Tools, Concepts*, 8.1.

The Bicycle Trip

This is a complicated distance-time graph, with a lot of information on it. It makes for a good discussion of what happened that day, which can be had after just reading the introductory paragraph and studying the graph. I have heard many entertaining explanations about what happened at the end of the day.

If you want more specific and more numerical questions, you can assign part or all of #1-8.

#9 frames an important discussion about distance-time graphs.

#10 can be used as a non-quiz assessment. I have sometimes used it as a take-home project.

The lesson is taken from *Algebra: Themes, Tools, Concepts*, 4.A.

Make These Designs

I have used this lesson as a way to reinforce and review y=mx+b.

Students don't have to do the problems in order. I require more accuracy from my stronger students, while from others I am happy if they get something close to the image on the worksheet. I give hints like "I don't see any lines in this area. How could you get there?" This is particularly needed to get some students to use negative slopes and slopes between -1 and 1.

I usually ask that they keep notes, so as to be able to write a report on how they made the designs.

The very last one, and the "vertical" lines design can't really be done well without a solid understanding of y=mx+b. I don't expect all the students to get those.

There is much more information about this activity on my Web site.

Doing Dishes

This activity is a comparison of linear and exponential growth. Students are guided through a process to come up with formulas for those.

Perimeter and Area Functions

In this lesson, students find formulas or simply patterns for perimeter and area. Most of the functions are linear, but the area patterns on the second page are quadratic. The formula for the double-staircases is easier to find than the one for the single staircases, and can serve as a hint for those.

The problems are adapted from *Algebra: Themes, Tools, Concepts*, 1.7 and 1.11.