Math Teachers Circles

Some Suggestions

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The Challenge

- Implementing lofty goals in actual sessions
- Handling a wide range of participants

Good Problems

Low Threshold

◊ The question is easily understood

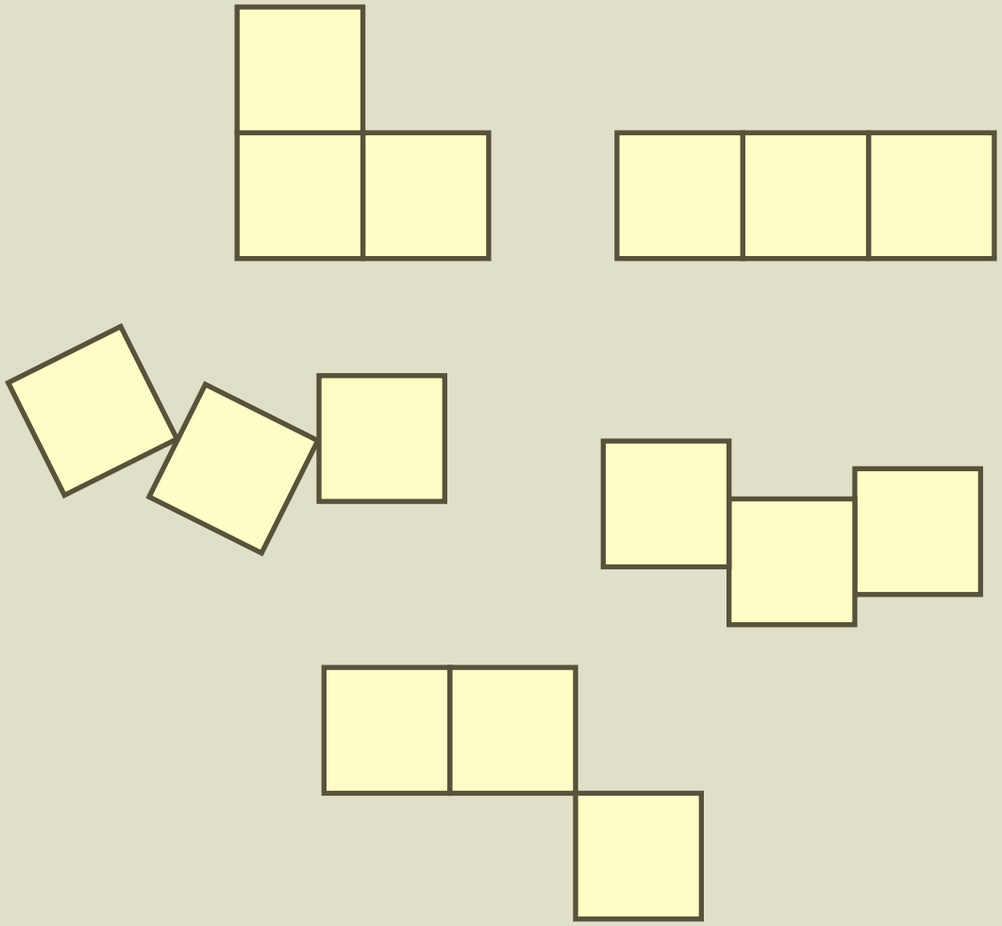
◊ There are few (or no) prerequisites

◊ Everyone can start exploring

Low Threshold

Example: polyomino perimeter

Domino



Trominoes

Not polyominoes

Low Threshold

Example: polyomino perimeter

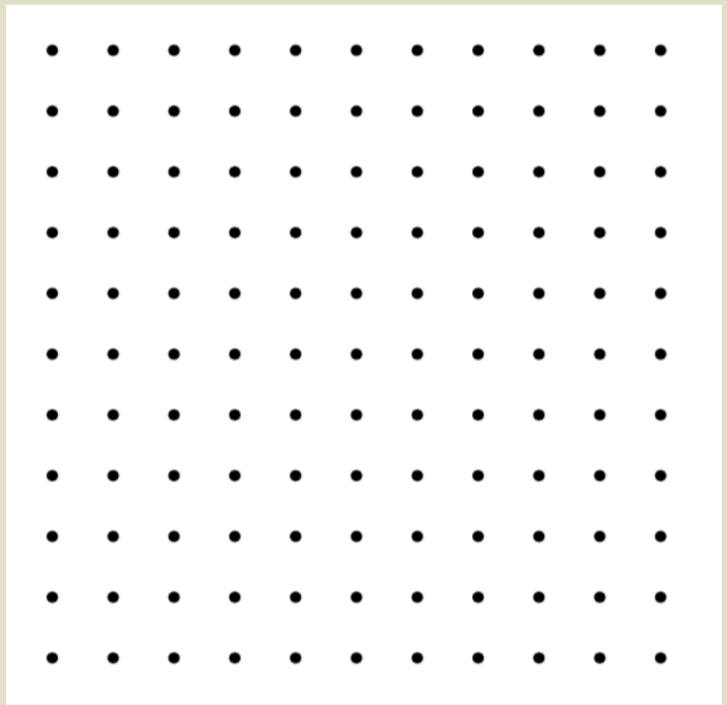
For a given area, what perimeters are possible?

Microworlds

Constrained environments that offer opportunities to engage with powerful ideas.

Microworlds

Example: the geoboard



Microworlds

Example: the geoboard

◊ How many points, so no three are collinear?

◊ Find “unexpected” isosceles triangles

◊ Find all triangles with a given area

◊ Pick’s formula

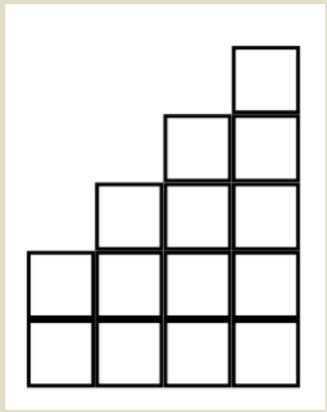
◊ etc.

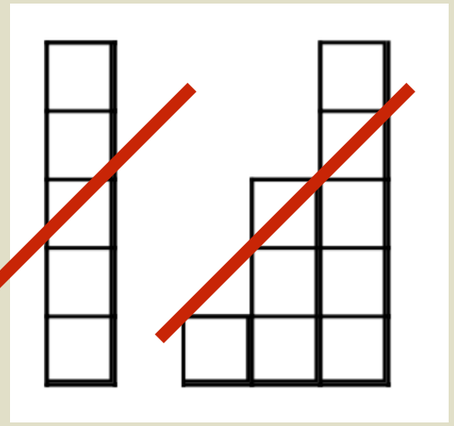
Multiple Paths

There is more than one way towards the solution.

Multiple Paths

Example: Staircases





Partial Solutions

There are interesting partial results to be found,
even if you don’t come up with a full solution

Partial Solutions

Example: Egyptian Fractions

Write each fraction as a sum of three or fewer unit fractions (fractions whose numerator is 1). One has been done for you. You don't have to do them in order. Don't use negative numbers.

$\frac{4}{3} =$

$\frac{4}{4} =$

$\frac{4}{5} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10}$

$\frac{4}{21} =$

$\frac{4}{22} =$

$\frac{4}{23} =$

$\frac{4}{39} =$

$\frac{4}{40} =$

$\frac{4}{41} =$

Extensions / Generalizations

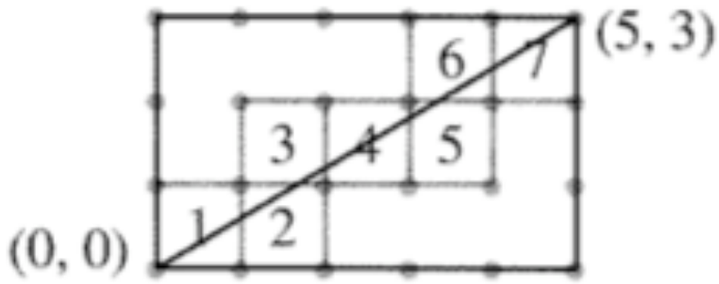
The problem can be extended or generalized.

Extensions / Generalizations

Example: Geoboard Diagonals

GEOBOARD DIAGONALS

If you connect (0, 0) to (5, 3) with a straight line, you go through seven unit squares.



14. Exploration If you connect (0, 0) to (p, q) with a straight line, how many unit squares do you go through? Experiment and look for patterns. (Assume p and q are positive whole numbers.) Keep a record of your work.

High Ceiling

- The problem should be interesting to you
- The problem should be group-worthy

Planning

- ◊ "Good problem" checklist
- ◊ Is the session "curricular"?
  If not, what is the "take-away"?
- ◊ Worksheet or not?
- ◊ Backup plan if things don't work out?

Problem Solving!

◊ Main goal: building a problem-solving culture

◊ Also: expanding participants' math knowledge

◊ Along the way: formal vs. informal times

Informal Time

◊ Participants work on the problem individually, or in pairs, or in small groups — as they choose

◊ This is what should take up the most time

◊ The challenge: people work at different rates

Etiquette

Do not *require* that people work together, instead encourage them to:

- ◊ ask for help if they need it
- ◊ offer help if they are asked
- ◊ share and discuss ideas

Arrange furniture to make that possible.

“If you have a solution…”

◊ find another one, or another path to this one

◊ extend / generalize the problem

◊ write up a clear explanation of your solution

"If you have a solution…"

◊ do not give it away!

◊ be *appropriately* helpful:

- ask questions

- give hints

(this applies to both leader and participants)

Teachers are Students!

◊ Make your expectations explicit

◊ Consider “visibly random” groups

◊ If participants’ focus drifts, bring them back in

◊ Direct intervention — not generic speeches

Formal Time

◊ This is a time for whole-group discussion.

◊ Needed if more than one group is totally stuck.

◊ Useful for sharing partial results

◊ No side conversations!

Transition to Formal Time

Use an agreed-upon signal

Sharing Results

◊ Choose groups or individuals who will share

◊ Sequence from least to most complete

◊ *Avoid repetition,* unless needed for understanding

Teaching?

◊ Yes, but mostly through questions

◊ The challenge: involving everyone

Teaching

Get responses from all:

◊ votes

◊ gestures

◊ writing

Teaching

Good questions:

◊ why?

◊ how do we know?

Not as good:

◊ yes or no?

◊ does everyone get it?

Teaching

To increase participation:

◊ wait, count

◊ be alert to gender, race, etc.

Helpful prompts:

◊ tell your neighbor

◊ restate what X said

Teaching

Praise:

◊ participation

◊ risk-taking

◊ problem-posing

Not so much:

◊ correct answers, which are their own reward

Teaching

Handling wrong answers:

◊ poker face

◊ write many answers

◊ “this is the right answer to…”

◊ “choose someone to help you”

Teaching

The punch line / big idea:

◊ is clear if the problem is curricular

◊ if not:

- what is it an instance of?

- how is it related to other math?

There is no one way

These are suggestions, not rules.
Much depends on:

- presenter personality
- nature of problem
- group dynamics
- etc.

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